Blind Pilot Decontamination

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HARP

Massive MIMO

Massive MIMO mimics the idea of spread spectrum.

- Spread spectrum:
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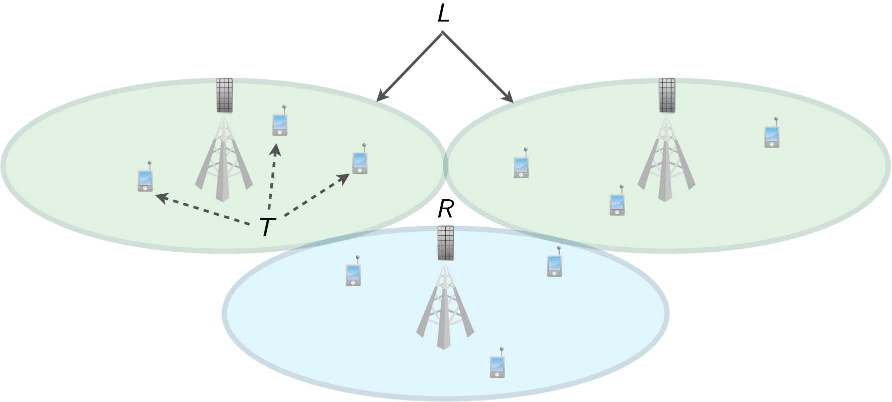
Both systems can operate in arbitrarily strong noise and interference.

Uplink (Reverse Link) System Model



$R \gg T$

$L \sim T$

$$\mathbf{Y} = \mathbf{HX} + \mathbf{Z}$$

Pilot Contamination

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How to estimate a massive MIMO channel appropriately?

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How to find the interference subspace or its orthogonal complement?

Matched Filter Projection

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- We get

$$\mathbf{m} = \underset{\mathbf{m}_0}{\operatorname{argmax}} \frac{||\mathbf{m}_0^\dagger \mathbf{Y}||^2}{||\mathbf{m}_0||^2} = \underset{\mathbf{m}_0}{\operatorname{argmax}} \frac{\mathbf{m}_0^\dagger \mathbf{Y}\mathbf{Y}^\dagger \mathbf{m}_0}{\mathbf{m}_0^\dagger \mathbf{m}_0}$$

  is that eigenvector of $\mathbf{Y}\mathbf{Y}^\dagger$ that corresponds to the largest eigenvalue.

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$$\mathbf{Y}' = \mathbf{M}^\dagger \mathbf{Y}$$

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We have utilized the array gain without estimating the channel.

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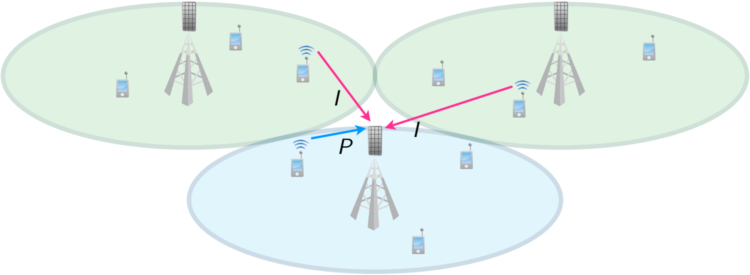
How to distinguish the signal of interest from interference?

Power Controlled Hand-Off

Consider power-controlled hand-off and perfect received power control.



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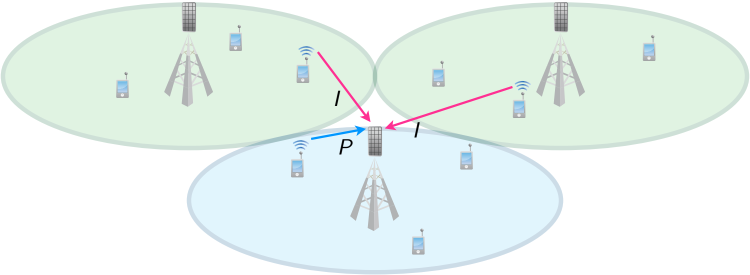

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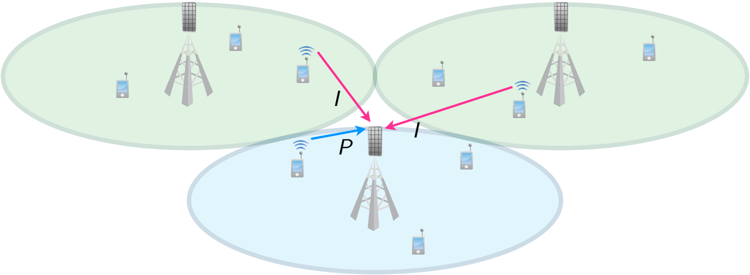

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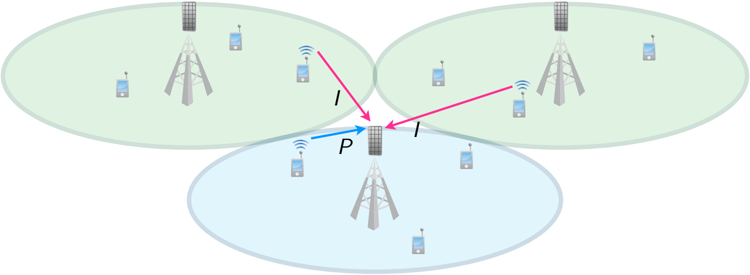

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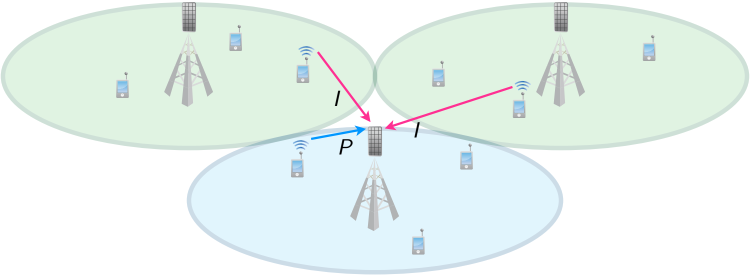

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What if the load is small, but not vanishing?

Asymptotic Eigenvalue Distribution

The exact *asymptotic* eigenvalue distribution can be given implicitly in terms of its Stieltjes transform

$$\mathrm{G}(s) = \int \frac{\mathrm{dP}(x)}{x - s}.$$

For an iid. channel, we find

$$\begin{aligned} s\mathrm{G}(s) + 1 = &- \frac{PTC\alpha\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)}{\alpha\kappa - PTC\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)} \\ &- \int \frac{xLTC\alpha\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)\,\mathrm{dP_I}(x)}{\alpha\kappa\,\mathrm{p_I}(x) - x\,TC\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)} \\ &- \frac{WC\alpha\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)}{\kappa} \end{aligned}$$

with W denoting the noise power, $\kappa = \frac{C}{R}$, and $\mathrm{P_I}(x)$ denoting the power distribution of the interference.

Asymptotic Eigenvalue Distribution

Assuming that all LT interferers have power I, i.e. $\mathrm{p_I}(x) = \delta(x - I)$, the fixed-point equation for the Stieltjes transform simplifies to

$$\begin{aligned} s\mathrm{G}(s) + 1 = &- \frac{PTC\alpha\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)}{\alpha\kappa - PTC\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)} \\ &- \frac{ILTC\alpha\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)}{\alpha\kappa - ITC\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)} \\ &- \frac{WC\alpha\big(s\mathrm{G}(s) + 1 - \kappa\big)\mathrm{G}(s)}{\kappa} \end{aligned}$$

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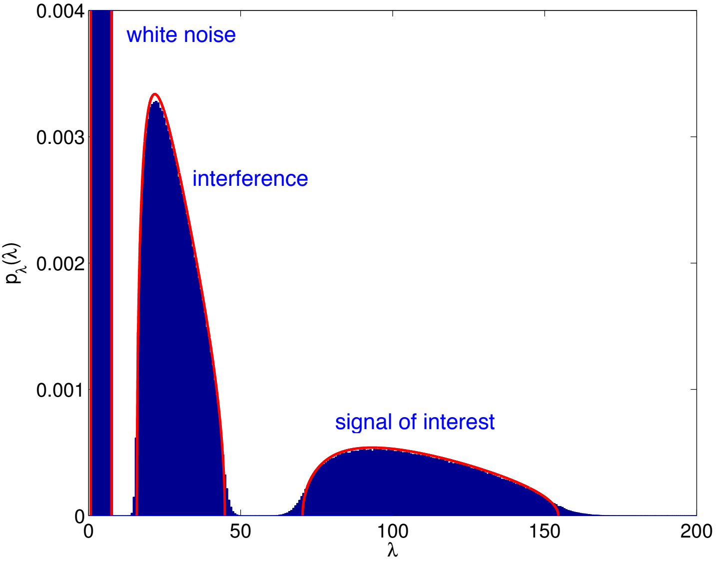
Empirical Eigenvalue Distribution



$$R = 300$$
$$T = 10$$
$$C = 1000$$
$$L = 2$$
$$W = 1000$$
$$P = 100$$
$$I = 25$$

Uplink vs. Downlink

We have detected the uplink data without estimating the full channel.

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Uplink vs. Downlink

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1. We use time-division duplex.
2. We project the received signal $\mathbf{Y}$ onto the orthogonal complement of the interference.
3. We use all uplink data to estimate the downlink (forward link) channel to high accuracy.

Eigenvalue Spread

Assume an i.i.d. channel matrix and $R \gg T \to \infty$.

The eigenvalues of the signal of interest are confined in an interval centered at the received power P with width

$$4P\sqrt{\frac{T}{R} + \frac{T}{C}}.$$

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$$4P\sqrt{\frac{T}{R}+\frac{T}{C}}.$$

The eigenvalues of the interference are confined in an interval centered at the interference power I with width

$$4I\sqrt{\frac{LT}{R}+\frac{LT}{C}}$$

where L denotes the number of interfering cells.

For massive MIMO, the two widths are quite small.

Eigenvalue Separation

The two intervals do not overlap if

$$\frac{P}{I} > \frac{1 + 2\sqrt{\frac{LT}{R} + \frac{LT}{C}}}{1 - 2\sqrt{\frac{T}{R} + \frac{T}{C}}}.$$

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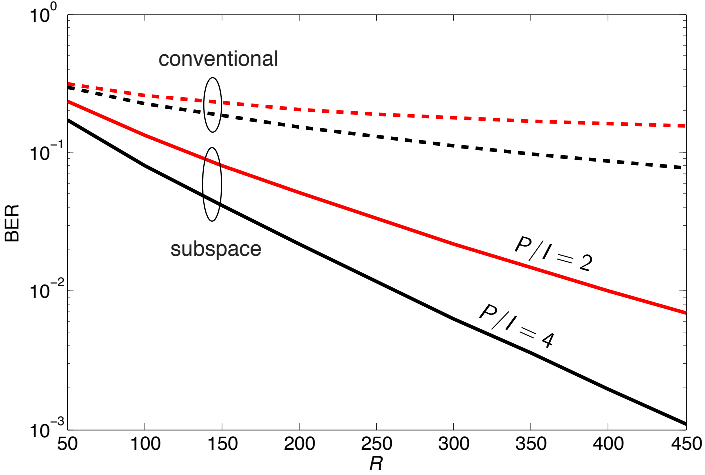
For finite number of receive antennas, the interval boundaries are not sharp, but have exponentially decaying tails.

BER vs. Array Size



$T = 3$

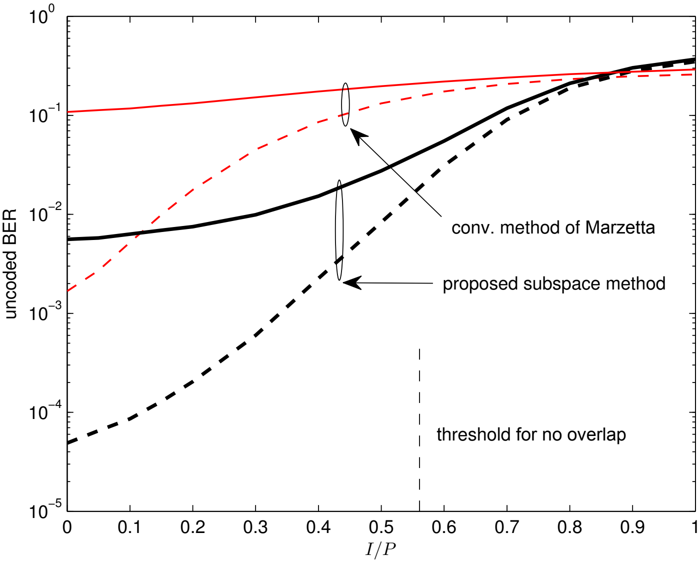
$C = 1000$

$L = 2$

$\mathrm{SNR} = -10\mathrm{dB}$

1 pilot symbol per transmit antenna and cell

BER vs. Power Margin



$R = 200$
$T = 2$
$C = 400$
$L = 2$
$W = 1$
$P = 0.1$

1 (–) or 10 (- -) pilot symbols per transmit antenna and cell

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Pro: A sufficient power margin can be established (with high probability).
Con: Users at cell boundaries may suffer from reduced data rate.

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- Pilot decontamination based on power control works well under the simulated conditions.
- The algorithm requires real-time eigenvalue or singular value decompositions.

Literature

- U. Madhow,"Blind adaptive interference suppression for direct sequence CDMA," Proc. of the IEEE, vol. 86, no. 10, pp. 2049–2069, Oct. 1998.
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