

When Users Interfere with Protocols

Prospect Theory in Wireless Networks

WINLAB 

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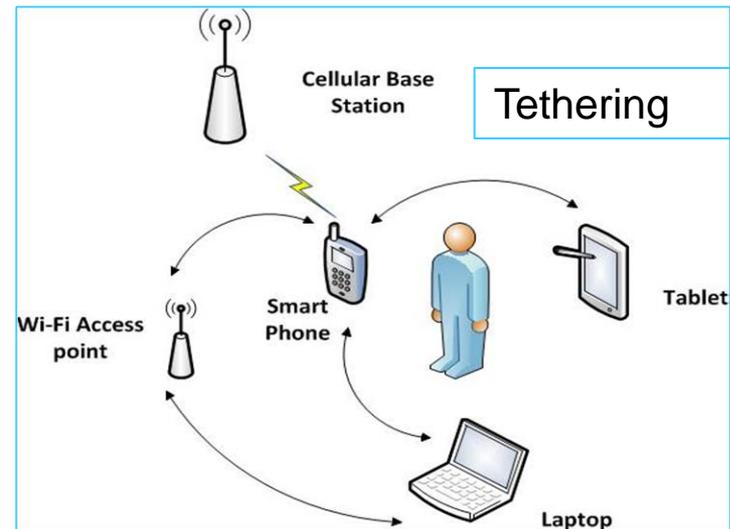
Motivation: Engineered System Design

- Current radio technologies and associated communication protocols are still mostly agnostic to the decision-making of end-users
 - “Engineered System Design” where underlying algorithms/protocols designed based on precepts of Expected Utility Theory (EUT)
 - Radio resource management algorithms and protocols are the result of optimization strategies under the framework of EUT

- Expected Utility Theory (EUT)
 - Alternatives with uncertainty are valued as their mathematical expectation
 - However, violations to it are constantly observed in real-life

Wireless: Increased End-User Influence

- End-users can influence system performance
- Cognitive radio, smart phone applications and user interfaces
 - ❑ Allow end users (people) greater degree of freedom to control devices
 - ❑ Impact underlying algorithms design and system performance
 - ❑ Example: user modifying radio cards and underlying protocols
 - ❑ Example: devices with flexible user interfaces
 - ❑ Example: end-user actions in response to link conditions, pricing



Prospect Theory: An Alternative to Expected Utility Theory

- Prospect L : a contract yields M outcomes, e.g., $\{o_1, \dots, o_M\}$, each occurring with probability p_i
 - How to value a prospect?

Expected Utility Theory (EUT)

- Proposed by Bernoulli, developed by Von Neumann, Morgenstern, others
- Game Theory heavily depends on it
 - E.g. game theoretic models in radio resource management
- Value of a prospect is estimated as the mathematical expectation of values of possible outcomes
- However, violations to EUT have constantly been observed in real-life decision-making

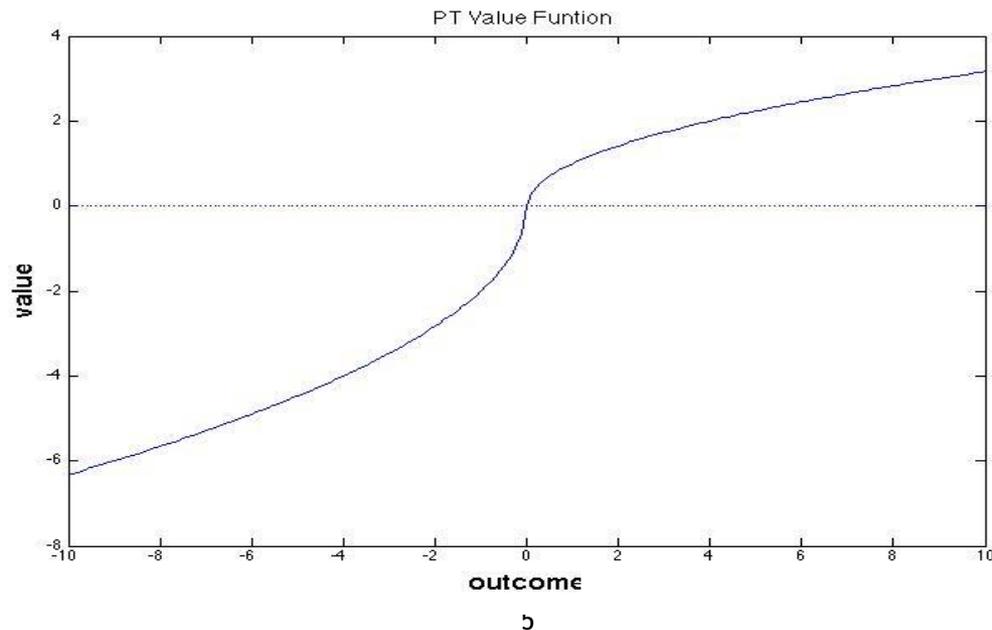
Prospect Theory (PT)

- Proposed by Kahneman and Tversky
- A better theory in describing people's real life decisions facing alternatives with risk
- Able to successfully explain the observed violations to EUT
- People use subjective probability to weigh values of outcomes
- People value outcomes in terms of relative gains or losses rather than final asset position

Prospect Theory: An Alternative to Expected Utility Theory

- Framing Effect

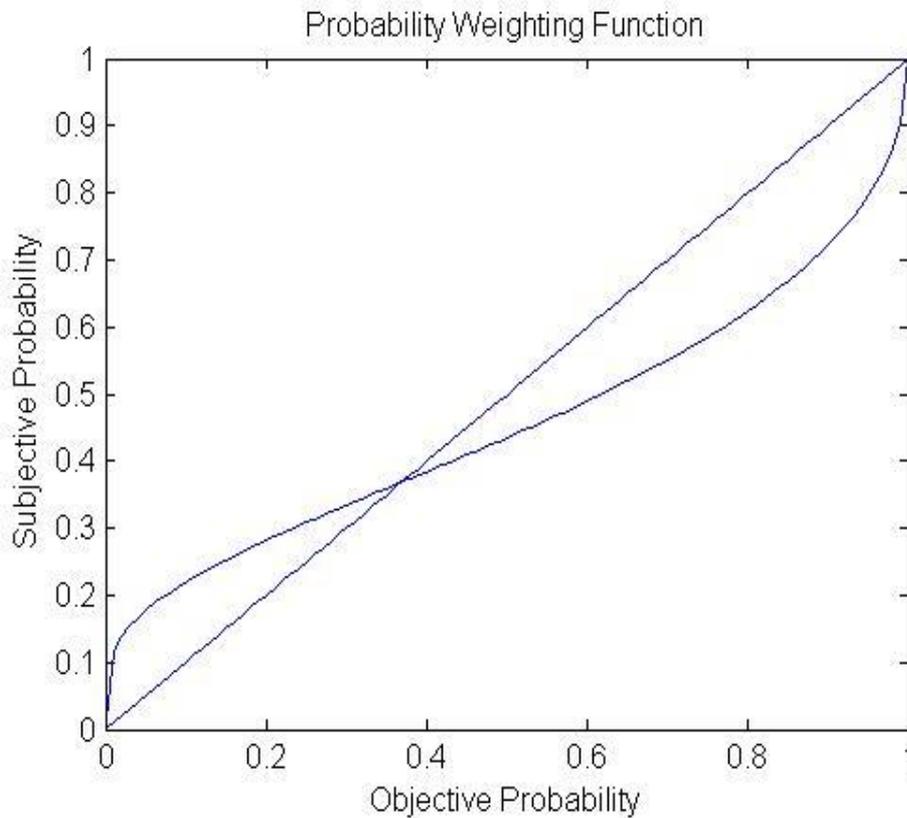
- People evaluate outcomes in terms of relative gains and losses regarding a reference point rather than the final asset position
- People's value function of outcomes is concave in gains and convex in losses
- Losses usually “loom larger” than gains



Prospect Theory: An Alternative to Expected Utility Theory

- Probability Weighting Effect

- People “nonlinearly transform” objective probabilities to subjective probabilities



- “Overweigh” low probabilities
- “Underweigh” moderate and high probabilities
- E.g. Asymmetrically reflected at $\frac{1}{e}$, i.e., $w\left(\frac{1}{e}\right) = 1/e$
- Concave in $\left[0, \frac{1}{e}\right]$, convex in $\left[\frac{1}{e}, 1\right]$
- People are able to objectively evaluate certainty, i.e.,
- $w(0) = 0 \quad w(1) = 1$

$$w(p) = \exp(-(-\ln p)^a), 0 < a \in 1$$

a characterizes deviation from EUT

Prospect Theory: Valuation of a Prospect

- Expected Utility Theory (EUT)

$$u^{EUT}(L) = \sum_{i=1, \dots, M} p_i v^{EUT}(o_i)$$

- Expectation of values of all possible outcomes

- Prospect Theory (PT)

“The Psychophysics of Chance”

$$u^{PT}(L) = \sum_{i=1, \dots, M} w(p_i) v^{PT}(o_i)$$

Probability Weighting Effect

Framing Effect

When EUT Fails, PT Explains

- A variation of Allais' paradox

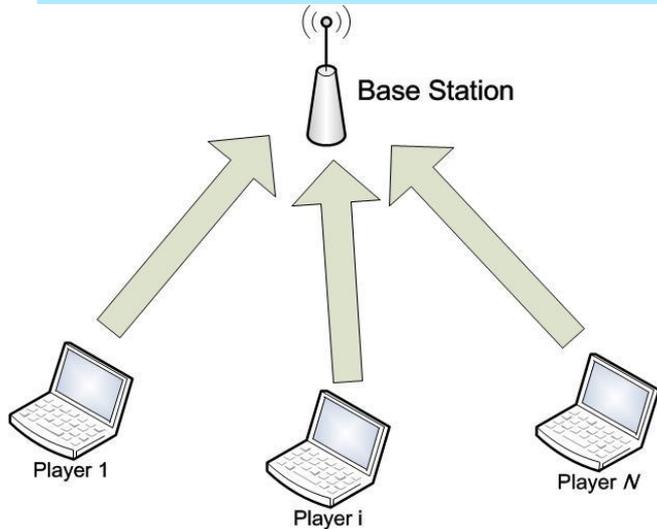
AN EXAMPLE OF EUT VIOLATION

Problem \ Prospect	A	B
1	\$2500 with probability 0.33 \$2400 with probability 0.66 \$0 with probability 0.01	\$2400 with certainty
2	\$2500 with probability 0.33 \$0 with probability 0.67	\$2400 with probability 0.34 \$0 with probability 0.66

- 61% respondents choose 1B and 2A
 - Under EUT,
 - 1B implies $0.34v^{EUT}(2400) > 0.33v^{EUT}(2500)$
 - 2A implies $0.34v^{EUT}(2400) < 0.33v^{EUT}(2500)$
 - Under PT with $\alpha = 0.5$ and linear value function with zero as the reference point, the two choices established simultaneously

Paradox

Toy Problem: Wireless Random Access



- A set of N selfish players accessing the same base station
- A time-slotted and synchronous system
- Each player has a saturated queue of packets

- In a time slot, a player can either transmit or wait, $a_i \in A_i = \{t, nt\}$

□ $t = \text{transmit}$ $nt = \text{NOT transmit}$

- Pure strategy profile: $\mathbf{a} = \{a_1, a_2, \dots, a_N\}$

- Collection of pure strategy profiles:

□ $\mathbf{A} = A_1 \times A_2 \times \dots \times A_N$

A Wireless Random Access Game

- If a player transmits
 - A successful transmission: obtains a unit throughput reward c_i and incurs a unit energy cost e_i
 - A failed transmission: incurs a unit delay penalty d_i and a unit energy cost e_i
- If a player waits: incurs a unit delay penalty d_i
- For both PT and EUT, we assume players use same value function
 - linear in unit throughput reward, delay penalty and energy cost with reference point zero
- Fix a pure strategy profile $\mathbf{a} = \{a_1, \dots, a_N\}$, a player evaluates the possible outcomes as

$$v_i | \mathbf{a} = \begin{cases} p_{i|\mathcal{J}(\mathbf{a})}(c_i - e_i) + (1 - p_{i|\mathcal{J}(\mathbf{a})})(-e_i - d_i) & \text{if } a_i = t \\ -d_i & \text{if } a_i = nt \end{cases}$$

A Wireless Random Access Game: Utility Functions

- Under Expected Utility Theory

$$u_i^{EUT}(\mathbf{p}) = \sum_{\mathbf{a} \in \mathbf{A}} \left(\prod_{j \in \mathcal{J}(\mathbf{a})} p_j \prod_{k \notin \mathcal{J}(\mathbf{a})} (1 - p_k) v_{i|\mathbf{a}} \right)$$

p_j - j - th player's transmission probability

- Objective expectation of values of all possible pure strategy profiles

- Under Prospect Theory

Strategy profile where the player transmits

Strategy profile where the player NOT transmit

$$u_i^{PT}(\mathbf{p}) = \sum_{\mathbf{a}_1 \in \mathbf{A}, a_{1i}=t} SP(\mathbf{a}_1) v_{i|\mathbf{a}_1} + \sum_{\mathbf{a}_2 \in \mathbf{A}, a_{2i}=nt} SP(\mathbf{a}_2) v_{i|\mathbf{a}_2}$$

- Values of all possible pure strategy profiles are weighed by subjective probabilities

Subjective transmission probability of player j viewed by player i

$$SP(\mathbf{a}_1) = p_i \prod_{j \in \mathcal{J}(\mathbf{a}_1) \setminus \{i\}} w_i(p_j) \prod_{k \in \mathcal{J}^c(\mathbf{a}_1)} w_i(1 - p_k)$$

$$SP(\mathbf{a}_2) = (1 - p_i) \prod_{j \in \mathcal{J}(\mathbf{a}_2)} w_i(p_j) \prod_{k \in \mathcal{J}^c(\mathbf{a}_2) \setminus \{i\}} w_i(1 - p_k)$$

Consequence of Deviation from EUT?

- 2-Player Heterogeneous Game
 - One PT player and one EUT player
- What impact does the PT player have compared to a 2-player homogeneous EUT game?
 - Performance change of the EUT player
 - Performance difference between PT and EUT player
 - Overall system performance
- Metrics Studied
 - Average Energy
 - Average Throughput
 - Average Delay

Utility Functions and Performance Metrics (Linear)

- Utility Functions $i = 1, 2$

- PT player:

$$u_i^{PT}(\mathbf{p}) = p_i w_i(p_j) v_{i|\{t,t\}} + p_i w_i(1 - p_j) v_{i|\{t,nt\}} + (1 - p_i)(-d_i)$$

- EUT player:

$$u_i^{EUT}(\mathbf{p}) = p_i p_j v_{i|\{t,t\}} + p_i(1 - p_j) v_{i|\{t,nt\}} + (1 - p_i)(-d_i)$$

- Communication Performance Measures $i = 1, 2$

$$T_i(\mathbf{p}) = c_i (p_i p_j p_{i|\{i,j\}} + p_i(1 - p_j) p_{i|\{i\}})$$

Throughput rewards

$$E_i(\mathbf{p}) = p_i e_i$$

Energy Costs

Delay Penalties

$$D_i(\mathbf{p}) = d_i (p_i p_j (1 - p_{i|\{i,j\}}) + p_i(1 - p_j)(1 - p_{i|\{i\}})) + (1 - p_i) d_i, \text{ for } i = 1, 2$$

Existence and Uniqueness of Mixed NE

- There exists a unique mixed NE for the Heterogeneous game if

$$v_{i|\{t,nt\}} > 0$$

- The value of a collision free transmission is “positive”

$$v_{i|\{t,t\}} < -d_i$$

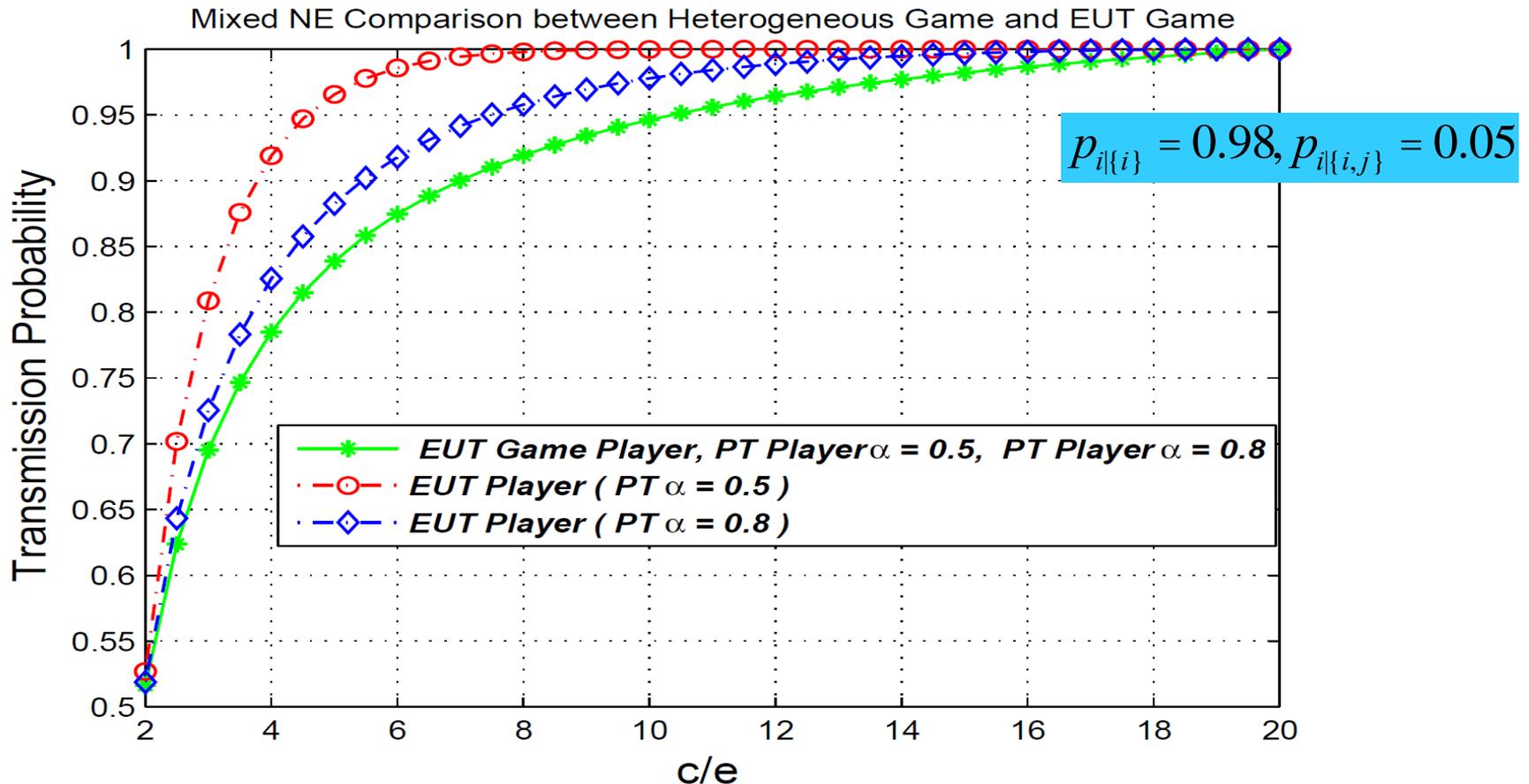
- A “negative” value results when there is a collision (simultaneous user transmission)
- The negative value is smaller than $-d_i$
 - d_i is the unit delay cost

Consequence of Deviation from EUT

Proven under mild conditions

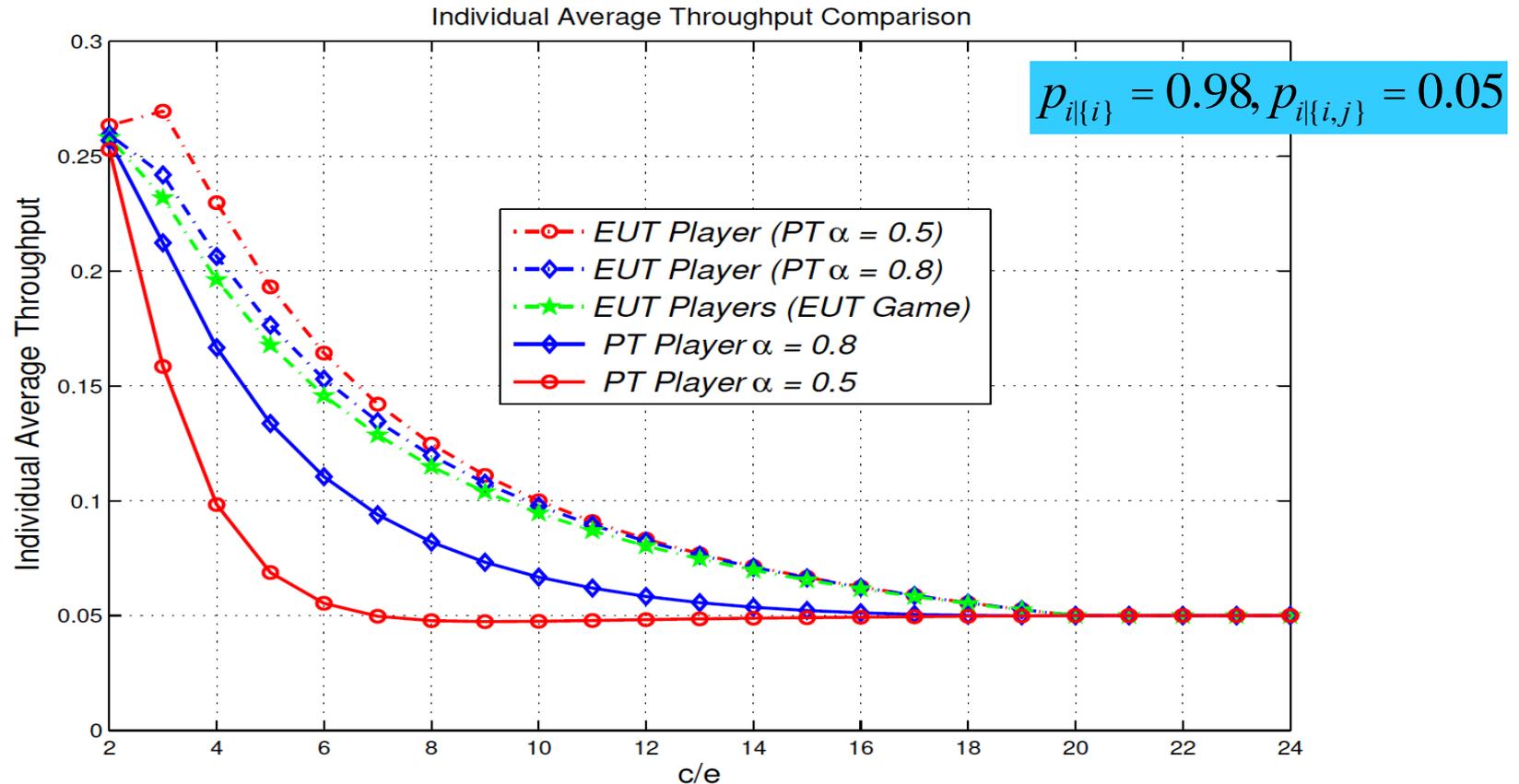
- *Consequence 1*: The PT player causes the EUT player
 - ❑ To gain higher average throughput
 - ❑ To experience lesser average delay
 - ❑ To incur higher average energy costs
- *Consequence 2*: The PT player
 - ❑ Achieves lesser average throughput
 - ❑ Experiences greater average delay
- *Consequence 3*: System level performance degraded
 - ❑ Lower total average throughput
 - ❑ Greater total average delay
 - ❑ Higher total average energy costs
- All the trends are exaggerated with lower a

Transmission Probability at Mixed NE (d=0)



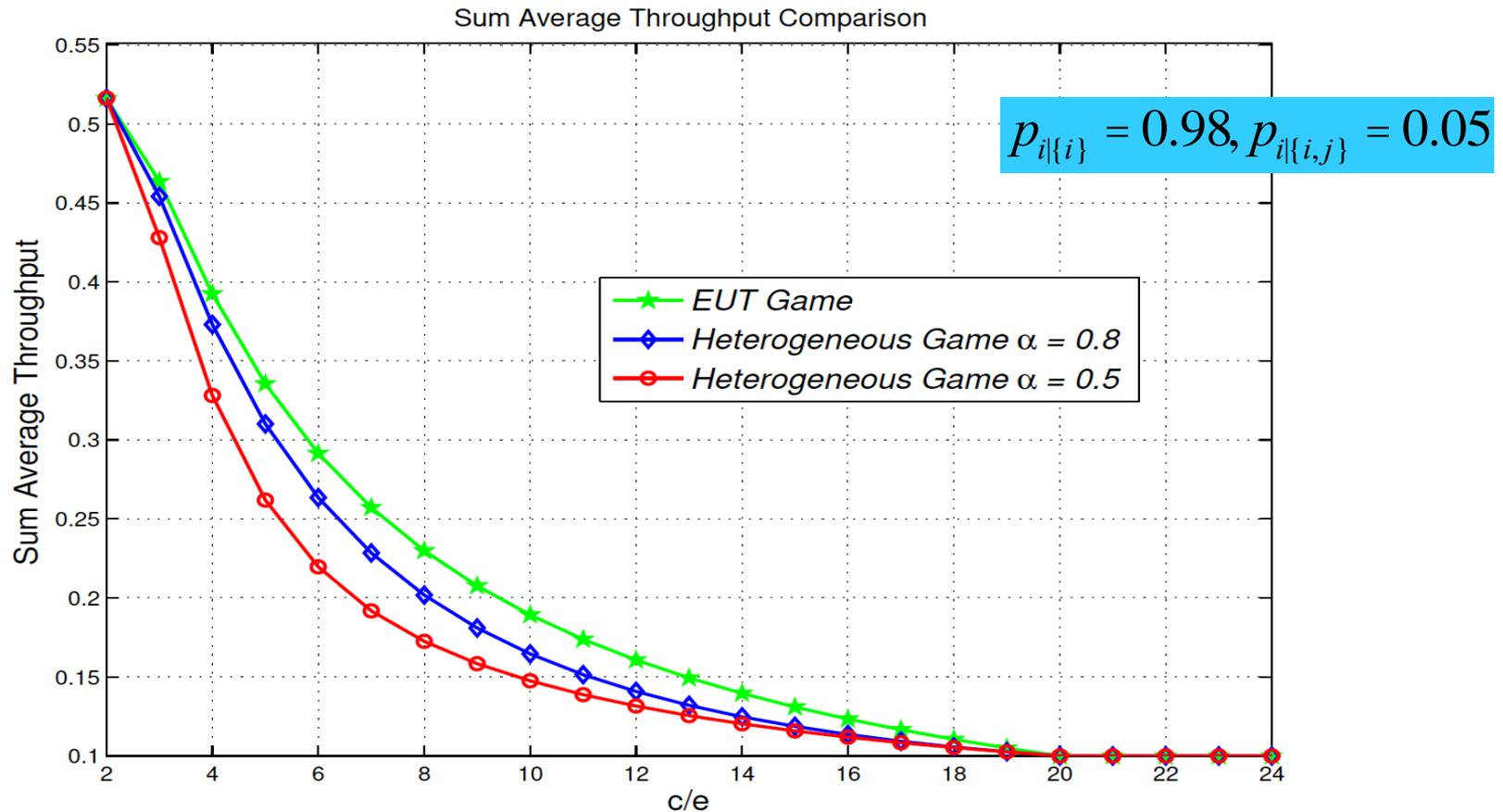
- EUT player if forced to transmit more aggressively
- If PT behavior is increasingly exaggerated, EUT player needs to be more aggressive

Individual Throughput Comparison (d=0)



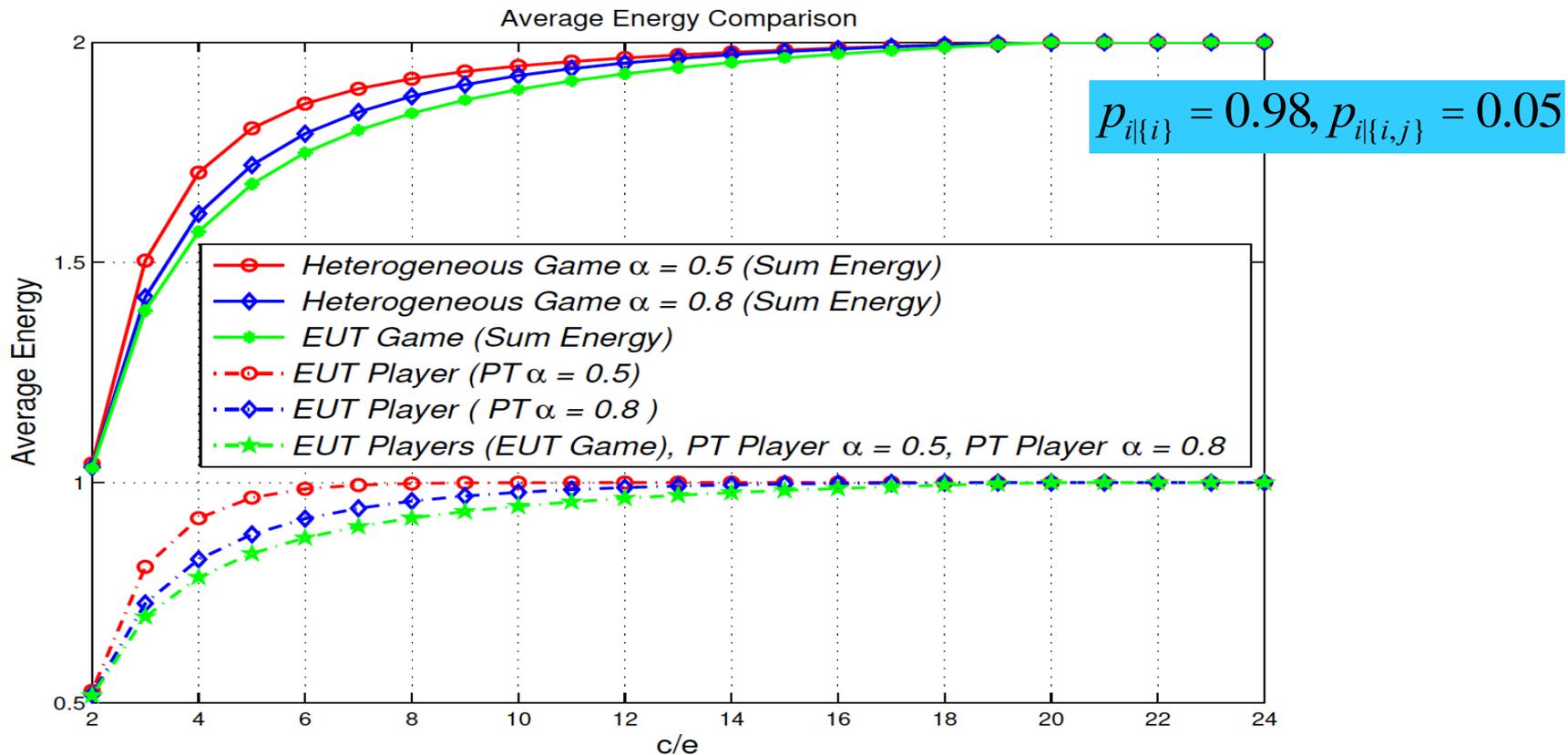
- ❑ Introduction of PT player makes EUT player gain more throughput rewards
- ❑ EUT player obtains more than PT player
- ❑ A more deviated PT player exaggerates the two trends

Sum Throughput Comparison (d=0)



- ❑ Total system throughput is degraded
- ❑ A more deviated PT player results in more severe degradation

Energy Costs Comparison (d=0)

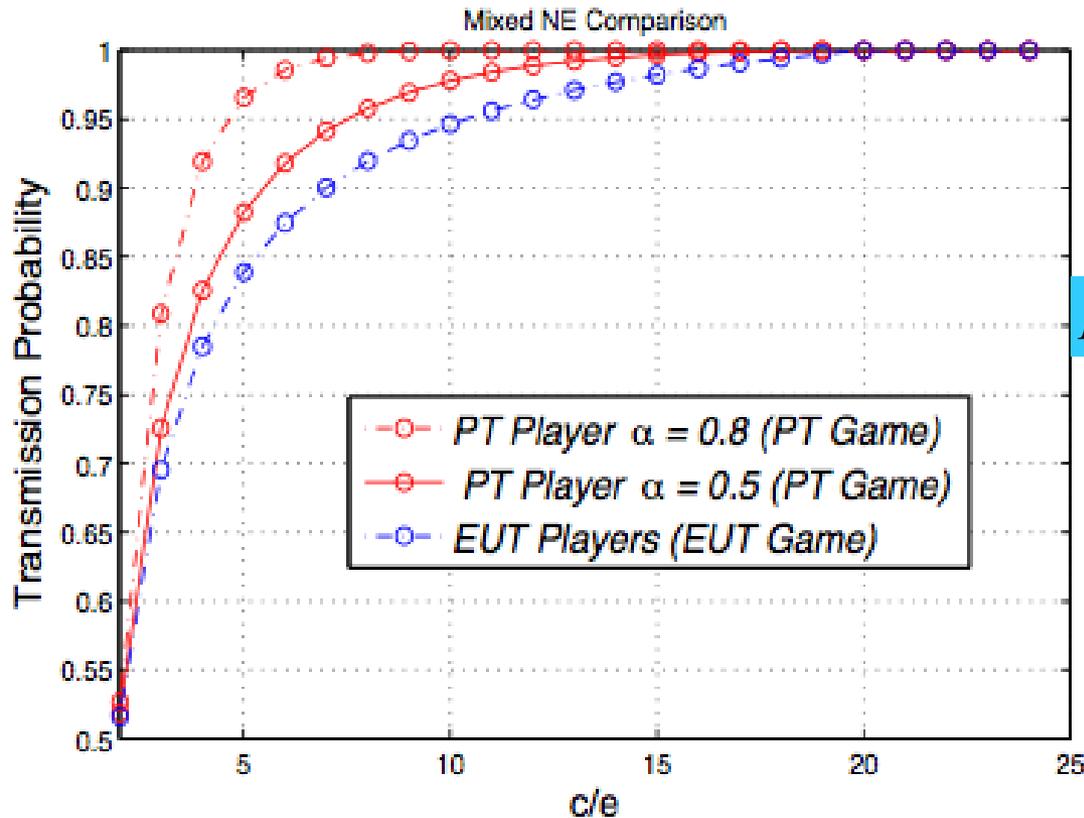


- ❑ Introduction of PT player causes EUT player to incur higher energy costs
- ❑ Introduction of PT player incurs higher system sum energy costs
- ❑ A more deviated PT player exaggerate the two trends

Homogeneous Game: Consequence of Deviation from EUT

- 2-Player Homogeneous Game
 - Two players are either both PT or both EUT
- *Consequence 4: System level performance degraded*
 - Lower total average throughput
 - Greater total average delay
 - Higher total average energy costs
- *Consequence 5: The PT player deviating less from EUT*
 - Achieves more average throughput
 - Suffers less average delay
 - But incurs more average energy cost

Transmission Probability at the mixed NE ($d = 0$)

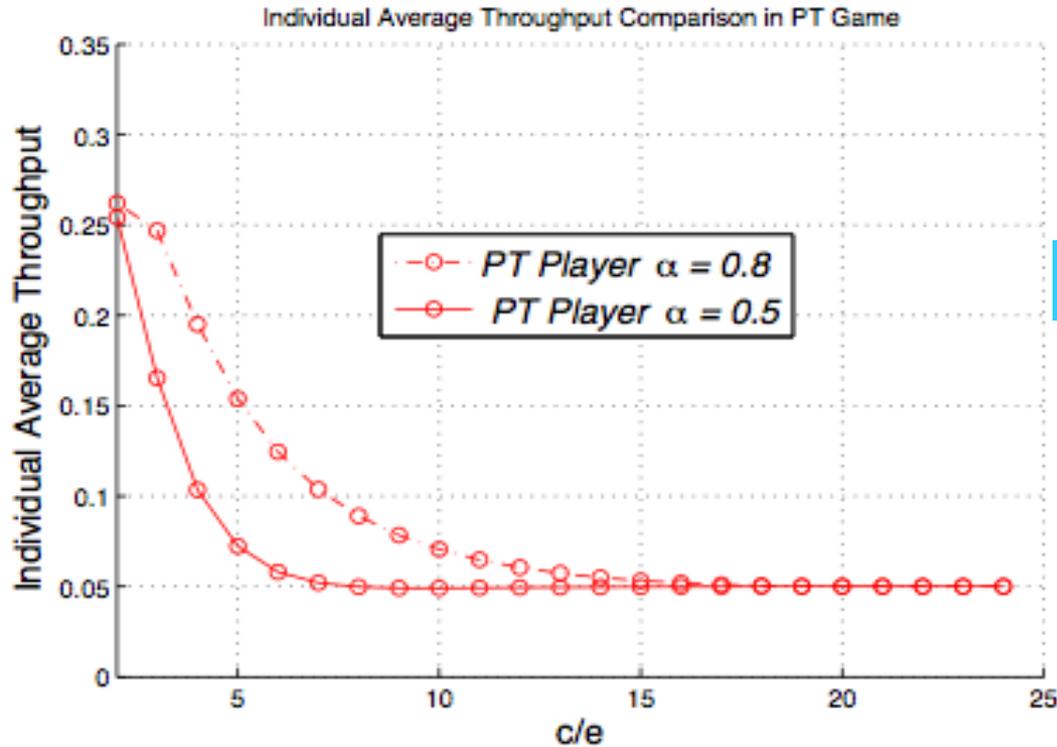


Homogeneous PT vs EUT Game

$$p_{i\{i\}} = 0.98, p_{i\{i,j\}} = 0.05$$

- PT players in PT game transmit more aggressively than the players of EUT game
- Within PT game, PT player deviates less from EUT transmits more aggressively

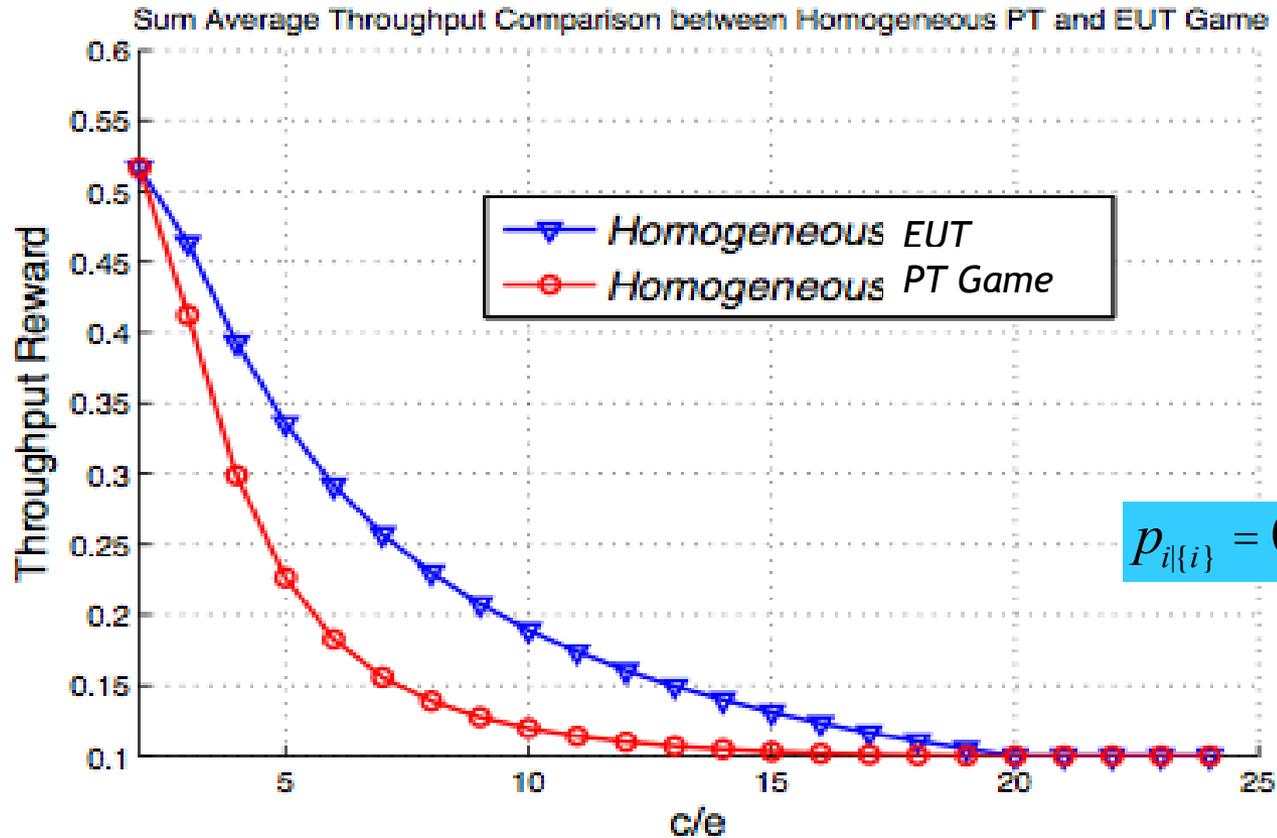
2-Player PT Game: Individual Average Throughput



$$p_{i\{i\}} = 0.98, p_{i\{i,j\}} = 0.05$$

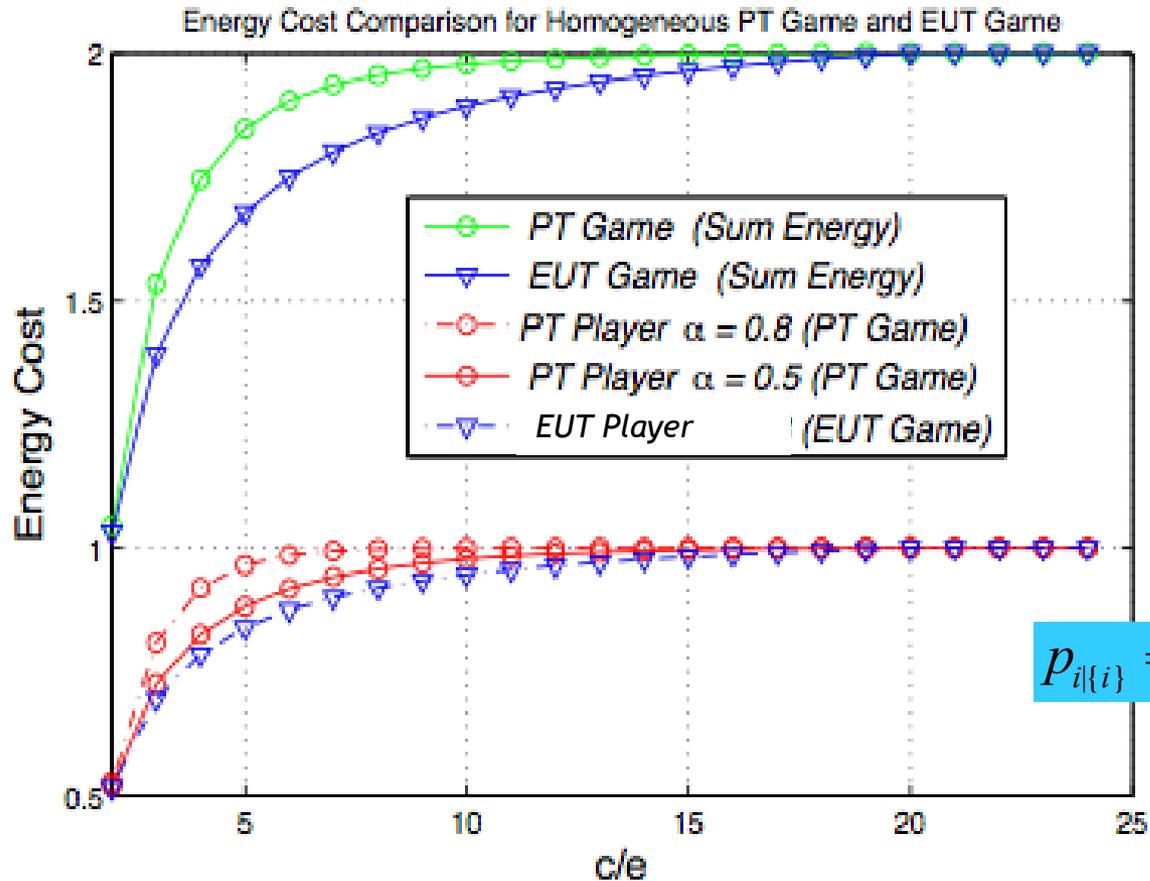
- The PT player that deviates less from EUT obtains more average throughput

PT vs. EUT Game: Sum Average Throughput



- ❑ Players in homogeneous PT game achieve less sum average throughput in the EUT game

PT vs. EUT Game: Energy Costs

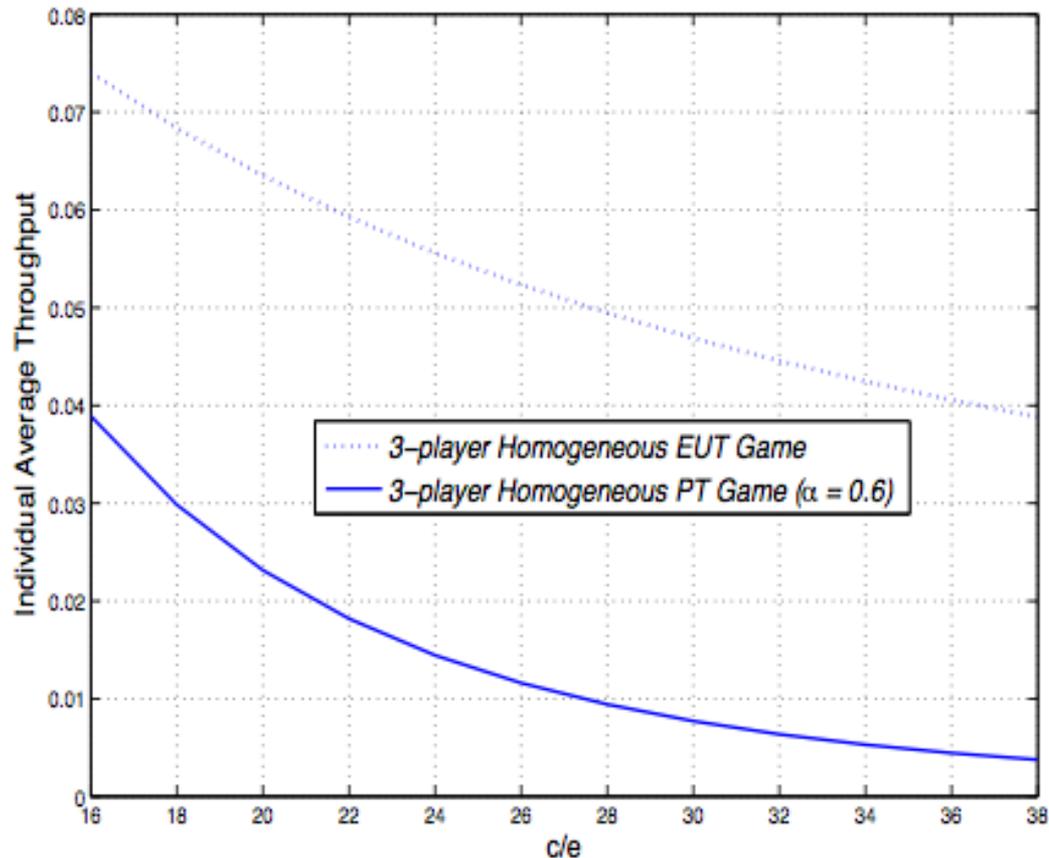


□ Players in PT game incur higher energy costs than players in EUT game

N-Player Homogeneous Game

- Symmetric: All players have identical utility functions and experience the same channel conditions
- Reflects a scenario where every player has a collective view of the set of players
 - “Collective” view of interference
 - Analyzing each of the other $N-1$ player’s utilities and actions is beyond a single user’s feasibility
- There exists a unique mixed NE for a symmetric N -player homogeneous game under mild conditions

3-Player Homogeneous Game: Average Throughput



- ❑ Fixed unit energy cost and unit delay penalty
- ❑ Degradation of average throughput

Prospect Theory: Wireless Applications

- Differentiated Pricing of Data Services for Network Congestion
 - User preferences, biases and perceived values
- SoNs - “organization/action” of people?
- Jamming in Wireless Networks
 - Biases and perceptions
- Robust Mechanisms for mitigating “user interference”
- Psychophysics experiments of wireless users
 - Design appropriate weighting and framing effects based on “wireless” experience

References

- T. Li and N. B. Mandayam, "**Prospects in a Wireless Random Access Game**" Proceedings of CISS'2012, Princeton NJ, March 2012
- T. Li and N. B. Mandayam, "**When Users Interfere with Protocols: Prospect Theory in Wireless Random Access**" under revision in IEEE Transactions in Wireless Communications, 2013