On the Cost of CSI Acquisition in Large MIMO Systems

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Many thanks to Ericsson Research Foundation!
CSI acquisition limits large-MIMO gains

Pilot symbols

TX

TX

RX

Capacity in the absence of a priori channel knowledge is the ultimate limit on the rate of reliable communication.
CSI acquisition limits large-MIMO gains

Pilot symbols

Capacity in the absence of a priori channel knowledge is the ultimate limit on the rate of reliable communication
Outline

1. Beyond the pre-log
2. Generic block-fading models
3. From asymptotics to finite-blocklength bounds
A simple channel model

<table>
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<tr>
<th>$h_n$</th>
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Constant block-memoryless Rayleigh-fading channel
Coherence time is the bottleneck

- MIMO input-output relation

\[ Y \leftarrow M_T \times S + X \leftarrow M_R \]

No closed-form expression available for \( C(\rho) \)

Pre-log \[ \chi = \lim_{\rho \to \infty} C(\rho) \log \rho = M^* \left( 1 - M^* L \right) \]

where \( M^* = \min \{ M_T, M_R, L/2 \} \)
Coherence time is the bottleneck

- MIMO input-output relation

\[ Y = S \times X + W \]

- No closed-form expression available for \( C'(\rho) \)

\[ \chi = \lim_{\rho \to \infty} \frac{1}{\rho} \log_{\rho} \left( M^* \left( 1 - \frac{M^*}{2} \right) \right) \]

where \( M^* = \min\{M_T, M_R, L/2\} \)
Coherence time is the bottleneck

- MIMO input-output relation

\[ Y = S \times X + W \]

- No closed-form expression available for \( C(\rho) \)

- Pre-log [Zheng & Tse, 2002]

\[ \chi = \lim_{\rho \to \infty} \frac{C(\rho)}{\log \rho} = M^* \left( 1 - \frac{M^*}{L} \right) \]

where \( M^* = \min\{M_T, M_R, L/2\} \)
The underlying geometry: $M_T = M_R = M$

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\chi = M \left(1 - \frac{M}{L}\right)
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$$\chi = M \left( 1 - \frac{M}{L} \right)$$

Communications on the Grassmannian manifold
Geometry suggests a signaling scheme

- **Uniform distribution** on the Grassmannian
  \[ X = \sqrt{L\rho} \ U \]

- **U**: (truncated) **unitary** and **isotropically distributed**

- Unitary space-time modulation (**USTM**)
A conjecture

Case $L \geq M_T + M_R$ (“small MIMO”) [Zheng & Tse (IT 2002)]:

$$C(\rho) = R_{USTM}(\rho) + o(1)$$
A conjecture

Case $L \geq M_T + M_R$ (“small MIMO”) [Zheng & Tse (IT 2002)]:

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Conjecture for $L < M_T + M_R$ (“large MIMO”) [Zheng & Tse (IT 2002)]:

- USTM not $o(1)$-optimal
BSTM is the optimal distribution

[Yang, Durisi, Riegler (JSAC 2013)]

BSTM is $o(1)$-optimal when $L < M_T + M_R$ (large-MIMO)

- $X = DU$ with
- $U$ i.d. and unitary
- $D^2$ diagonal; contains the eigenvalues of a complex matrix-variate beta distributed matrix
Why is BSTM optimal? The SIMO case

Large MIMO $\Rightarrow L < 1 + M_R$
Why is BSTM optimal? The SIMO case

- Large MIMO $\Rightarrow L < 1 + M_R$
- USTM $\Rightarrow x$ i.d., $\|x\|^2 = L\rho$
Why is BSTM optimal? The SIMO case

Large MIMO $\Rightarrow L < 1 + M_R$

USTM $\Rightarrow \mathbf{x}$ i.d., $\|\mathbf{x}\|^2 = L\rho$

BSTM $\Rightarrow \mathbf{x}$ i.d., $\frac{L-1}{\rho LM_R}\|\mathbf{x}\|^2 \sim \text{Beta}(L - 1, M_R + 1 - L)$
Why is BSTM optimal? The SIMO case

- Large MIMO $\Rightarrow L < 1 + M_R$
- USTM $\Rightarrow x$ i.d., $\|x\|^2 = L\rho$
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$$I(x; Y) = h(Y) - h(Y \mid x)$$
Why is BSTM optimal? The SIMO case

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- USTM $\Rightarrow x$ i.d., $\|x\|^2 = L\rho$
- BSTM $\Rightarrow x$ i.d., $\frac{L-1}{\rho L M_R} \|x\|^2 \sim \text{Beta}(L - 1, M_R + 1 - L)$

$$I(x; Y) = h(Y) - h(Y | x)$$

$$\approx h(s \|x\|) + 2(L - 1 - M_R) \mathbb{E}[\log \|x\|] + \text{const}$$
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2. Generic block-fading models
3. From asymptotics to finite-blocklength bounds
The “generic” block-fading model

- Constant block-fading model for subchannel \((r, t)\)

\[
h_{r,t} = 1_L \cdot s_{r,t}, \quad s_{r,t} \sim \mathcal{CN}(0, 1)
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- A more accurate model for MIMO CP-OFDM systems

\[ h_{r,t} = z_{r,t} \cdot s_{r,t}, \quad s_{r,t} \sim \mathcal{CN}(0, 1) \]

\[ z_{r,t} \in \mathbb{C}^L \Rightarrow \text{Fourier transf. of power-delay profile} \]
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\(z_{r,t} \in \mathbb{C}^L \Rightarrow \) Fourier transf. of power-delay profile

- We assume that \(\{z_{r,t}\}\) are generic
Generic $\{z_{r,t}\}$ yield larger pre-log

[Riegler, Koliander, Durisi, Hlawatsch (ISIT 2013)]

- $\{z_{r,t}\}$ generic and $M_R > \frac{M_T(L-1)}{L-T}$ with $M_T < L/2$
Generic \( \{z_{r,t}\} \) yield larger pre-log

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- Then

\[
\chi_{\text{gen}} = M_T \left(1 - \frac{1}{L}\right)
\]
Generic $\{z_{r,t}\}$ yield larger pre-log

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- $\{z_{r,t}\}$ generic and $M_R > \frac{M_T(L-1)}{L-T}$ with $M_T < L/2$

- Then

$$\chi_{\text{gen}} = M_T \left(1 - \frac{1}{L}\right)$$

- Compare with constant block-fading model

$$\chi_{\text{const}} = M_T \left(1 - \frac{M_T}{L}\right)$$
Intuition behind pre-log increase:
\( M_R = 3, M_T = 2, L = 4 \)

- Constant block-fading: \( \chi_{\text{const}} = M_T \left( 1 - \frac{M_T}{L} \right) = 1 \)
Intuition behind pre-log increase:

\[ M_R = 3, \ M_T = 2, \ L = 4 \]

- **Constant block-fading:**
  \[ \chi_{\text{const}} = M_T \left( 1 - \frac{M_T}{L} \right) = 1 \]

- **Generic block-fading:**
  \[ \chi_{\text{gen}} = M_T \left( 1 - \frac{1}{L} \right) = \frac{3}{2} \]
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Lost in “asymptotia”?
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- capacity characterizations up to $o(1)$ yield tight bounds 😊
- pre-log sensitive to small changes in the channel model 😞
From asymptotia to tight bounds

[Yang, Durisi, Koch, Polyanskiy (ITW 2012)]

\[
\chi = 1 - \frac{1}{20} = 0.95
\]
From asymptotia to tight bounds

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**Replacements**

Outage capacity ($C_\epsilon$)

LTE-Advanced codes

Converse

Achievability

Normal Approximation

Blocklength, $n$

Rate, bits/ch. use

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G. Durisi

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Zero dispersion

- AWGN channel [Polyanskiy, Poor, Verdú (IT 2010)]

\[
R^*_{\text{awgn}}(n, \epsilon) = C_{\text{awgn}} - \sqrt{\frac{V}{n}} Q^{-1}(\epsilon) - O\left(\frac{\log n}{n}\right)
\]
Zero dispersion

- **AWGN channel** [Polyanskiy, Poor, Verdú (IT 2010)]

\[ R_{\text{awgn}}^*(n, \epsilon) = C_{\text{awgn}} - \sqrt{\frac{V}{n}} Q^{-1}(\epsilon) - O\left(\frac{\log n}{n}\right) \]

- **SISO quasi static** [Yang, Durisi, Koch, Polyanskiy (ISIT 2013)]

\[ \{R_{\text{csirt}}^*(n, \epsilon), R_{\text{no}}^*(n, \epsilon)\} = C_\epsilon - \sqrt[4]{\frac{1}{n}} - O\left(\frac{\log n}{n}\right) \]
Summary

Capacity without a-priori CSI
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Too conservative estimates?

- USTM $\Rightarrow$ BSTM
- $M (1 - \frac{M}{L}) \Rightarrow M (1 - \frac{1}{L})$
Summary

Capacity without a-priori CSI

Too conservative estimates?
- USTM $\Rightarrow$ BSTM
- $M \left( 1 - \frac{M}{L} \right) \Rightarrow M \left( 1 - \frac{1}{L} \right)$

From asymptotia to finite blocklength
Backup Slides
Gain of BSTM over USTM for large-MIMO systems

\[ R_{BSTM} - R_{USTM} \]

\[ \frac{R_{BSTM} - R_{USTM}}{R_{USTM}} \]

\[ M_T = \min\{M_R, L/2\} \]

\[ \rho = 30 \text{ dB} \]

\[ L = 10, 20, 50, 100 \]

\[ M_R = 10, 20, 50, 100 \]
Achievability for finite blocklength