

# Privacy, Anonymity, and Ambiguity in Social and Information Networks

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CTW 2013



# AOL Search Log Release (2006)

4417749	care packages	2006-03-02	09:19:32
4417749	movies for dogs	2006-03-02	09:24:14
4417749	blue book	2006-03-03	11:48:52
4417749	best dog for older owner	2006-03-06	11:48:24
4417749	best dog for older owner	2006-03-06	11:48:24
4417749	rescue of older dogs	2006-03-06	11:55:25
4417749	school supplies for the iraq children	2006-03-06	13:36:33
4417749	school supplies for the iraq children	2006-03-06	13:36:33
4417749	pine straw lilburn delivery	2006-03-06	18:35:02
4417749	pine straw delivery in gwinnett county	2006-03-06	18:36:35
4417749	landscapers in lilburn ga.	2006-03-06	18:37:26
4417749	pne straw in lilburn ga.	2006-03-06	18:38:19
4417749	pine straw in lilburn ga.	2006-03-06	18:38:27
4417749	gwinnett county yellow pages	2006-03-06	18:42:08

...



anonymized user ID

# User 4417749 Uncovered by New York Times

## ■ Searches:

- “Landscapers in Lilburn, Ga”
- “homes sold in shadow lake subdivision gwinnett county georgia”
- “jarrett t. arnold”, “jack t. arnold”

## ■ 4417749=Thelma Arnold

- 62 years old widow and dog owner
- home: Lilburn, GA

## ■ AOL press release:

- “There was no personally identifiable data provided by AOL with those records, but search queries themselves can sometimes include such information.”

## ■ Heads had to roll...

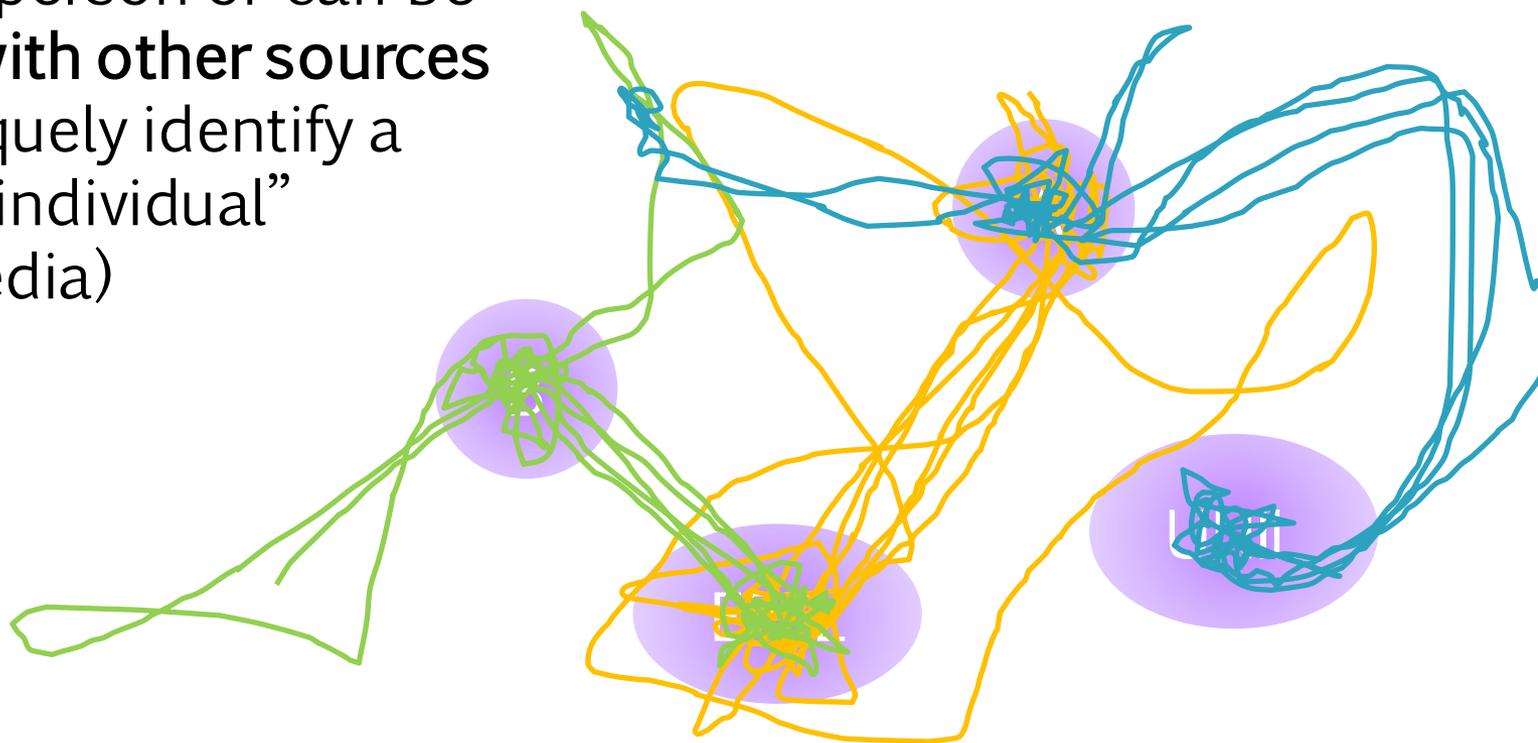
- AOL CTO Maureen Govern (+2 others) fired



# Privacy: Hard to Define

- **Personally identifiable information (PII):**
  - “information that can be used to uniquely identify, contact, or locate a single person or can be used **with other sources** to uniquely identify a single individual” (wikipedia)

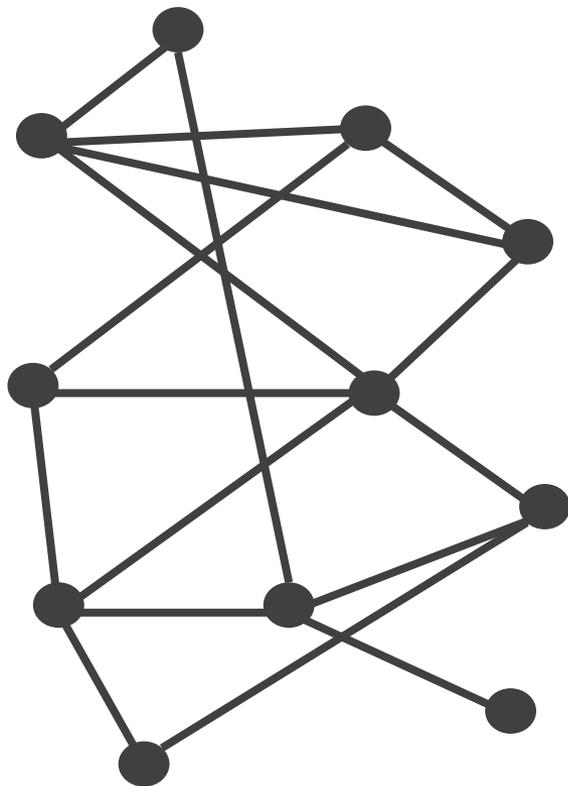
Name	Home	Work
Adam	A	EPFL
Barbara	B	EPFL
Carlos	A	UNIL



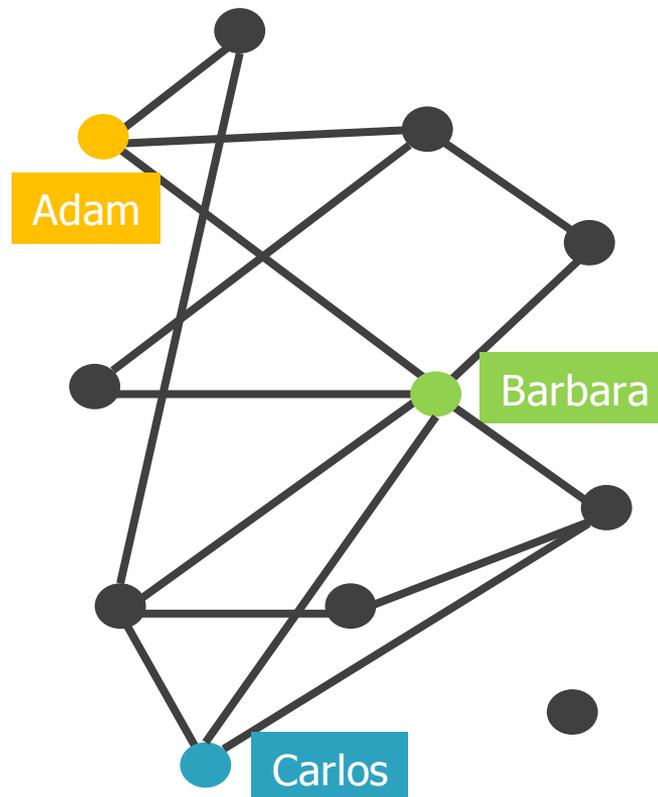
# Privacy of Networks

- **Adversary has:**
  - Anonymized network = unlabeled graph
  - Side information: subgraph; statistics on certain nodes; noisy version of whole network; ...

anonymized social network



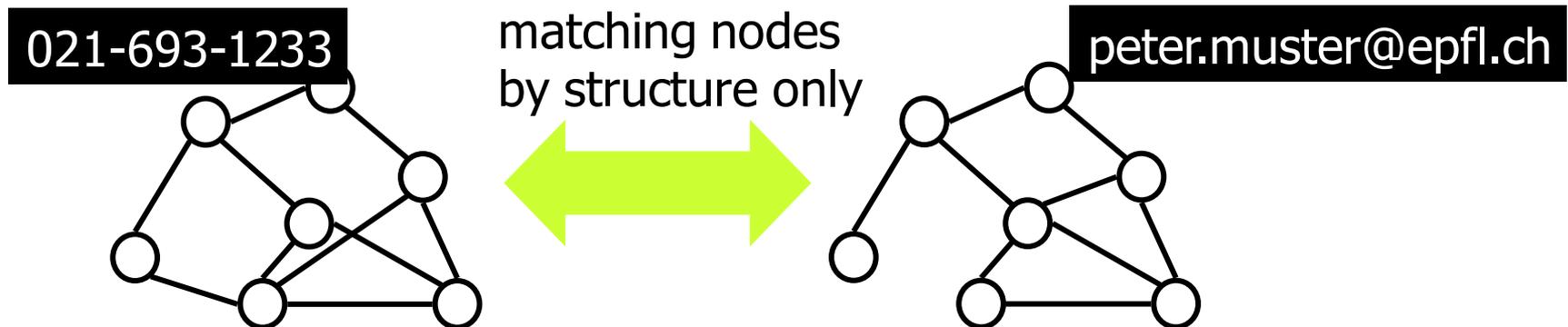
side information



# Graph Matching

- **Other applications:**

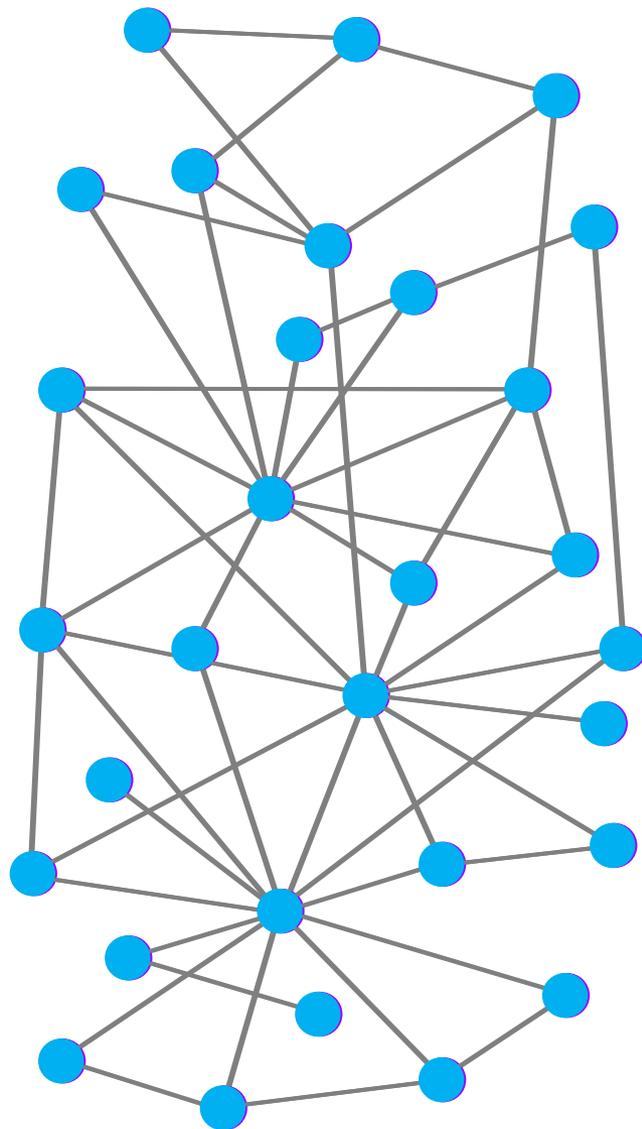
- Find overlap in networks:
  - Social networks from different domains & time slots
  - Identify viruses by function-call patterns
  - Computer vision: matching segment graphs for different viewing angles
- ...



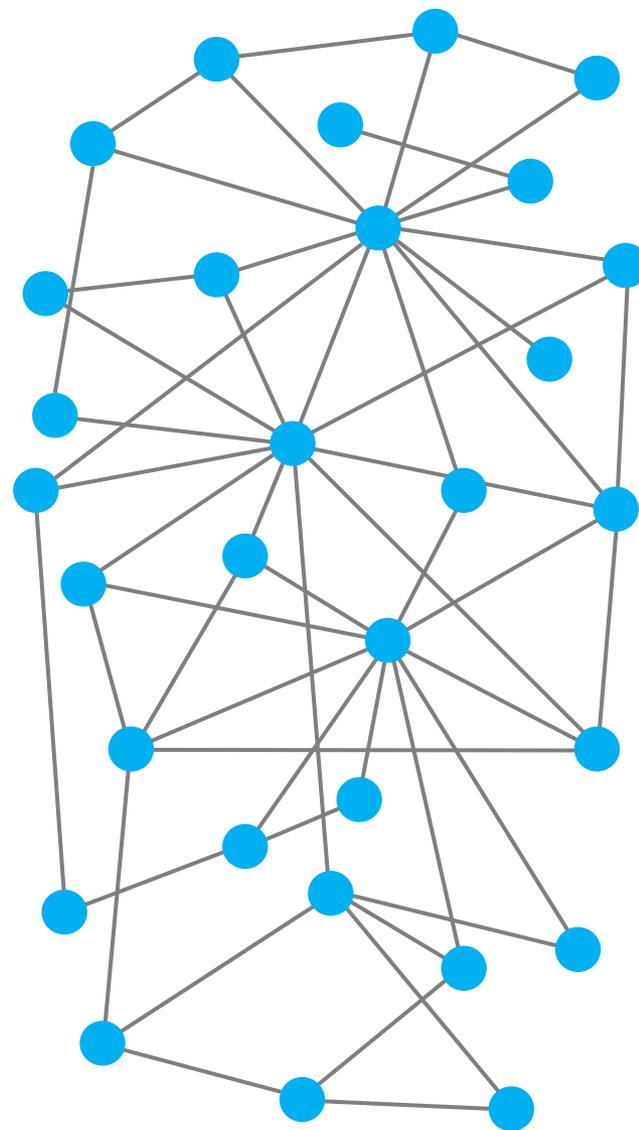
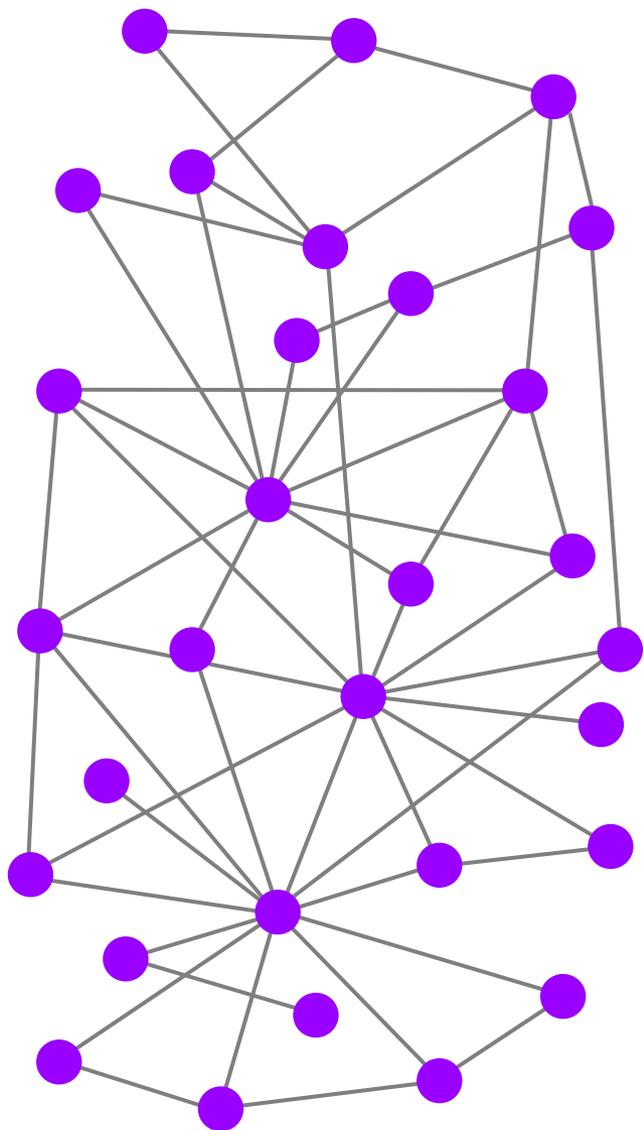
# No limits

Fundamental feasibility w/o side information, but with  $\infty$  time and memory

# Graph Matching



# Graph Matching with Noise



# Question

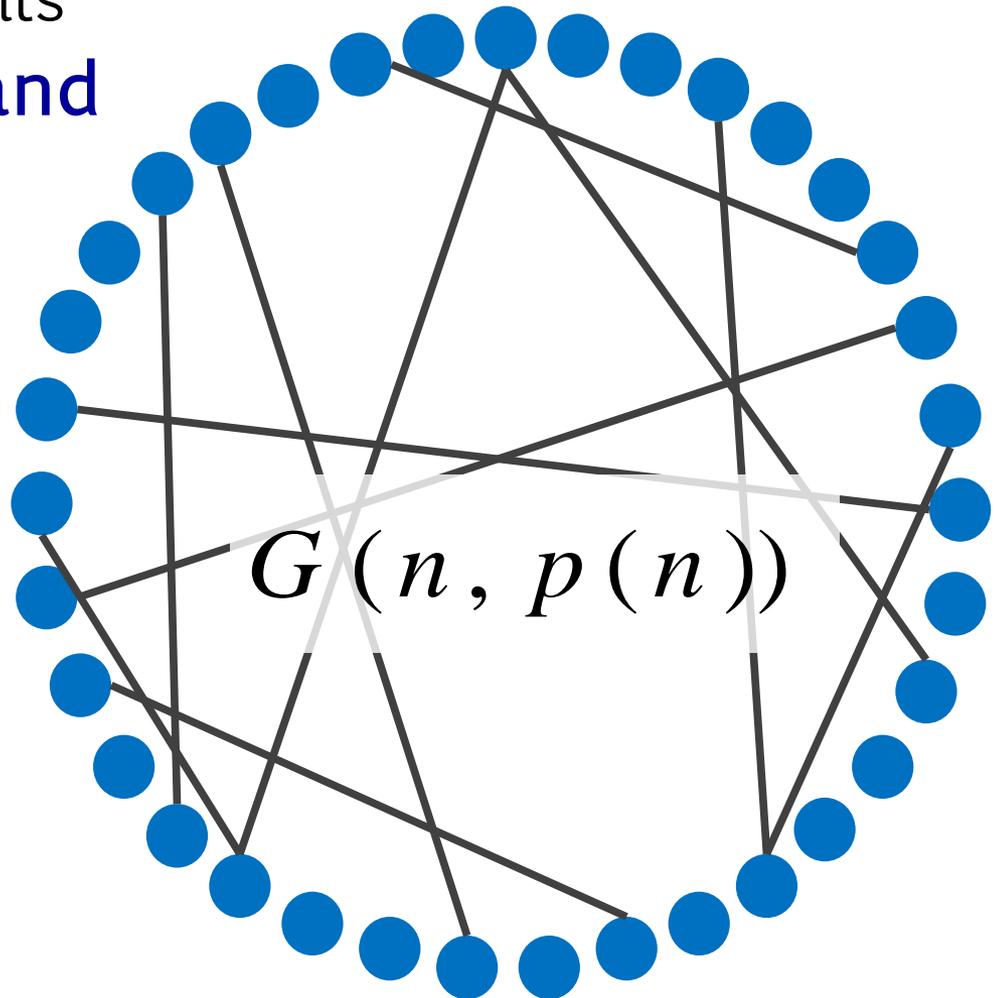
- Is it fundamentally hard or easy to match similar graphs by structure?
- Fundamental =
  - Information-theoretic: ignore computational & memory cost
  - Hard: in addition to second graph, no other side information
  - Demanding: want to match every vertex

# Random Graphs Instant Primer

- First published 1959 by Erdős & Rényi
  - Focus on existence results
- Large  $n$  asymptotics and phase transitions
  - Connectivity
  - Existence of subgraphs
  - Giant component
  - Chromatic number
  - Automorphism group
  - ...

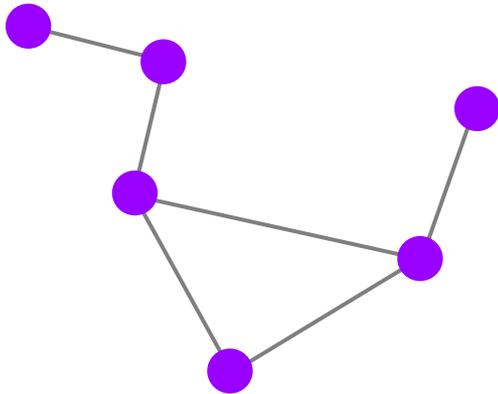
Threshold for  
asymmetry:

$$p = \log n / n$$



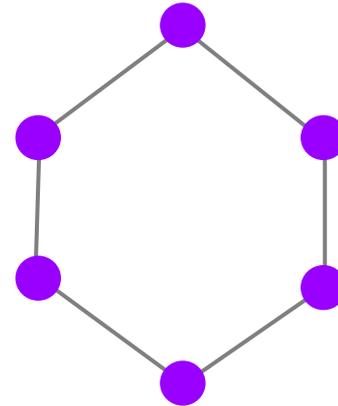
# Automorphism: Asymmetric vs Symmetric

Asymmetric



AuG = 1

Symmetric



AuG = 12

AuG = size of automorphism group

# $G(n, p; s)$ Sampling Model

Generator  $G = G(n, p)$

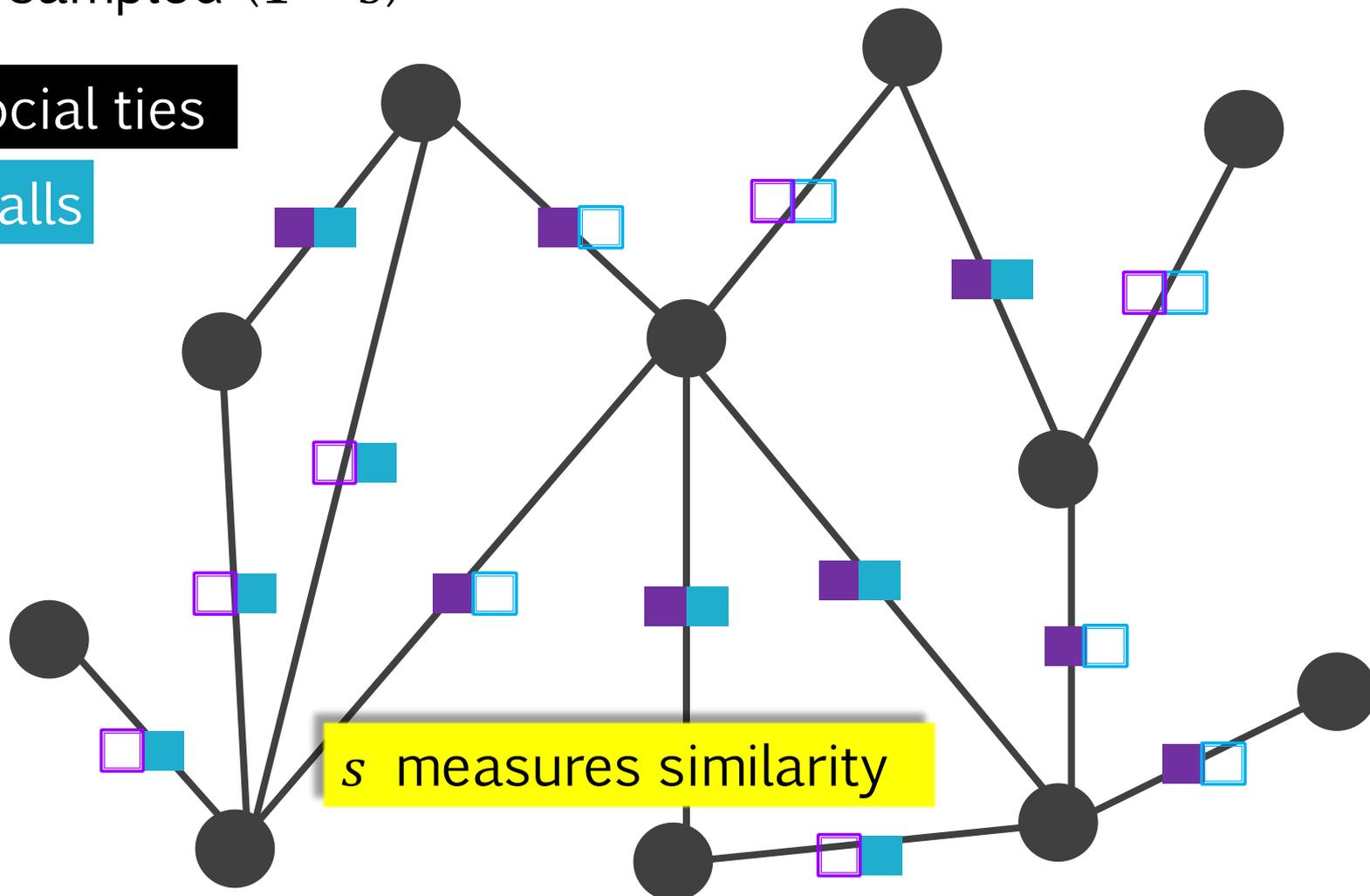
■ sampled ( $s$ )

□ not sampled ( $1 - s$ )

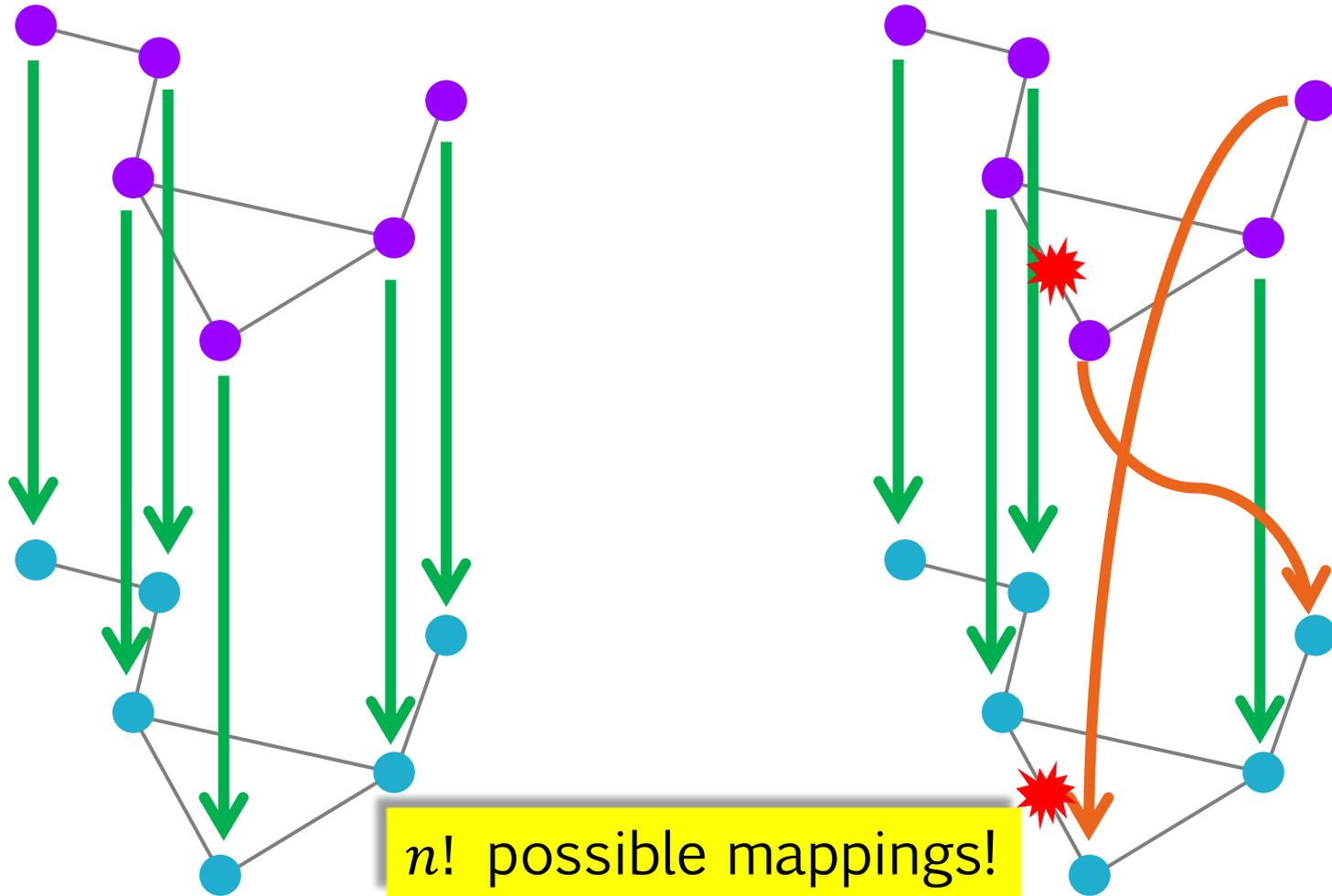
“real” social ties

phone calls

emails



# Mappings and Edge Mismatch



$$\Delta(\pi_0) = 0$$

$$\Delta(\pi) = 2$$

# Adversary Model

- **Assumption:**

- Attacker has infinite computational power
- Can try all possible mappings  $\pi$  and compute edge mismatch function  $\Delta(\pi)$

- **Question:**

- Are there conditions on  $p, s$  such that

$$P \{ \pi_0 \text{ unique min of } \Delta(\pi) \} \rightarrow 1$$

- If yes: adversary would be able to match vertex sets only through the structure of the two networks!

- **Note:**

- $G(n, p; s)$  model: statistically uniform, low clustering, degree distribution not skewed  $\rightarrow$  conjecture: harder than real networks

# Result: Difficult to Anonymize!

- **The**  $nps$ :  $E[\text{degree}]$  of  $G_{1,2}$  threshold for  $\text{aug}(G)=1$
- For the  $G(n,p;s)$  matching problem, if

$$\frac{nps}{2-s} = 8 \log n + \omega(1)$$

then the identity permutation minimizes  $\Delta(\cdot)$  a.a.s.

Penalty for difference  $G_1 - G_2$

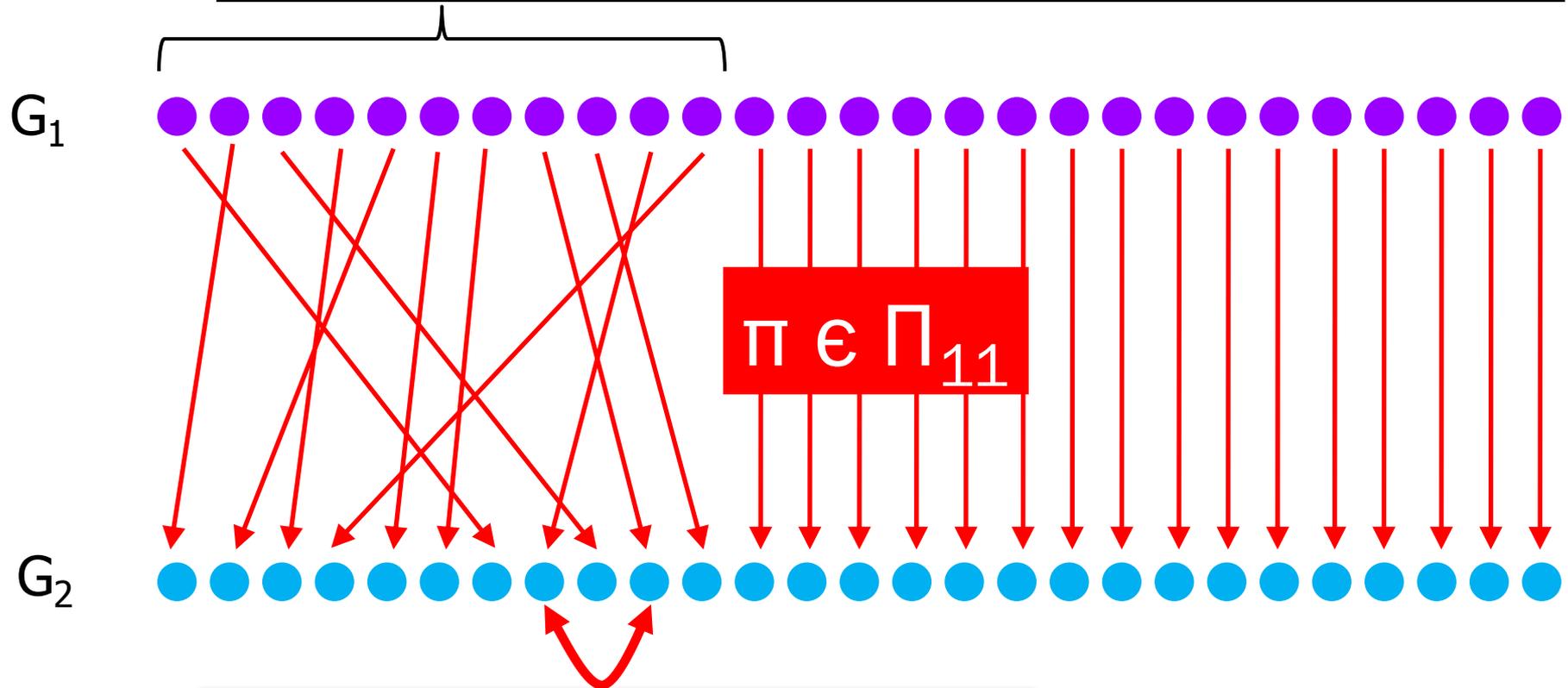
“growing slowly”

- **Interpretation: two pieces of bad/good news**
  - Surprisingly weak condition: degree growing faster than  $\sim \log n$  enough to break anonymity
  - Decrease with  $s$  only quadratic

# Proof Sketch

- Fix a particular map  $\pi$

$V_\pi$ : set of mismatched nodes under  $\pi$



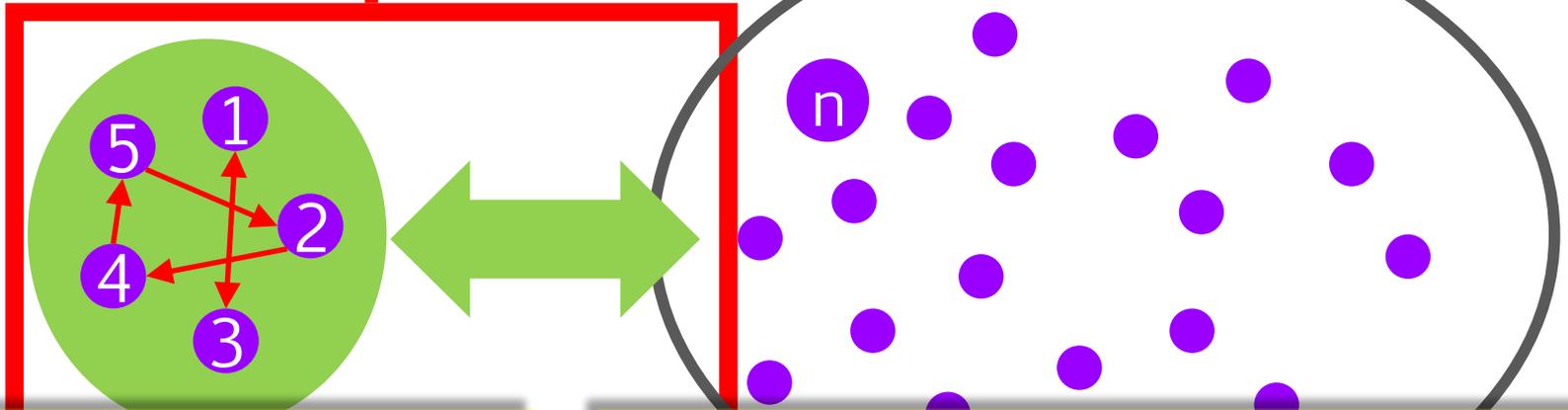
Transposition  $\Rightarrow$  invariant edge

# Proof Sketch

$E_\pi = V \times V_\pi$ :  
all the edges  
modified under  $\pi$

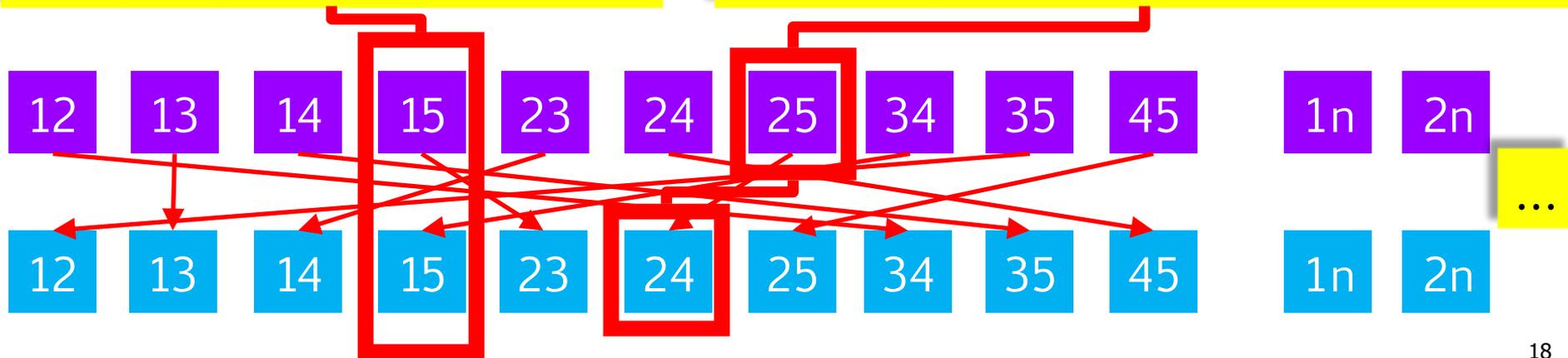
$V_\pi$ :  $k$  nodes

$n - k$  nodes



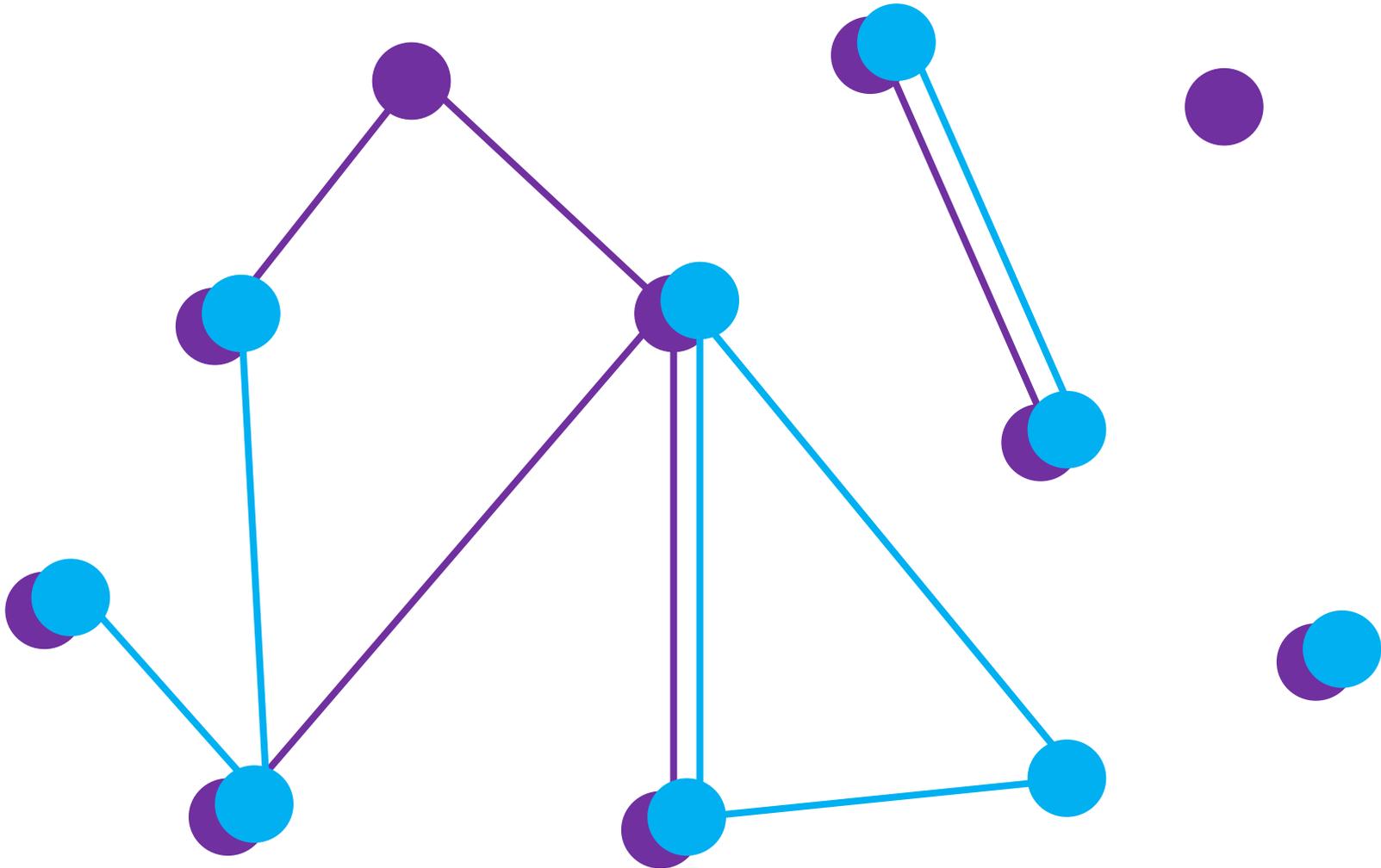
$\Delta_0$ : each edge contributes  
Bernoulli( $2ps(1 - s)$ ):  
sampling errors

$\Delta_\pi$ : each pair of edges contributes  
Bernoulli( $2ps(1 - ps)$ ):  
matching errors



# Extension: Node plus Edge Sampling

$G(n, p; s, t)$  matching problem



# Extension: $G(n, p; s, t)$ Matching Problem

- **Result:**
  - Dependence on  $n$  still the same:

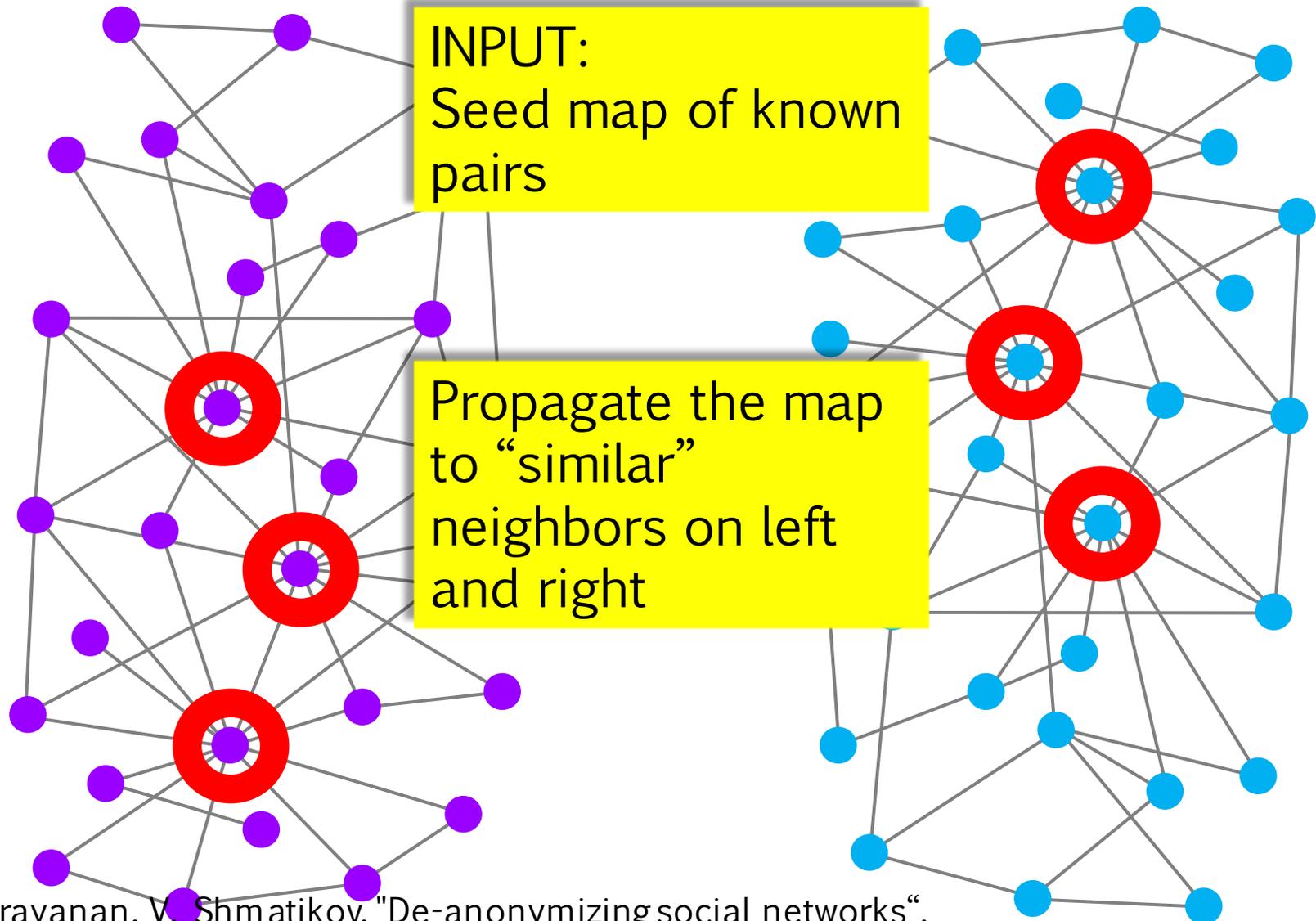
$$nps = c(s, t) \log n + \omega(1)$$

- Dependence on  $s$  and  $t$  less intuitive
- **Interpretation:**
  - Node mismatch does not help/hurt too much either

**Seeds =  
known matched pairs**

Phase transition, and an efficient &  
tractable matching algorithm...

# Map Propagation Heuristics

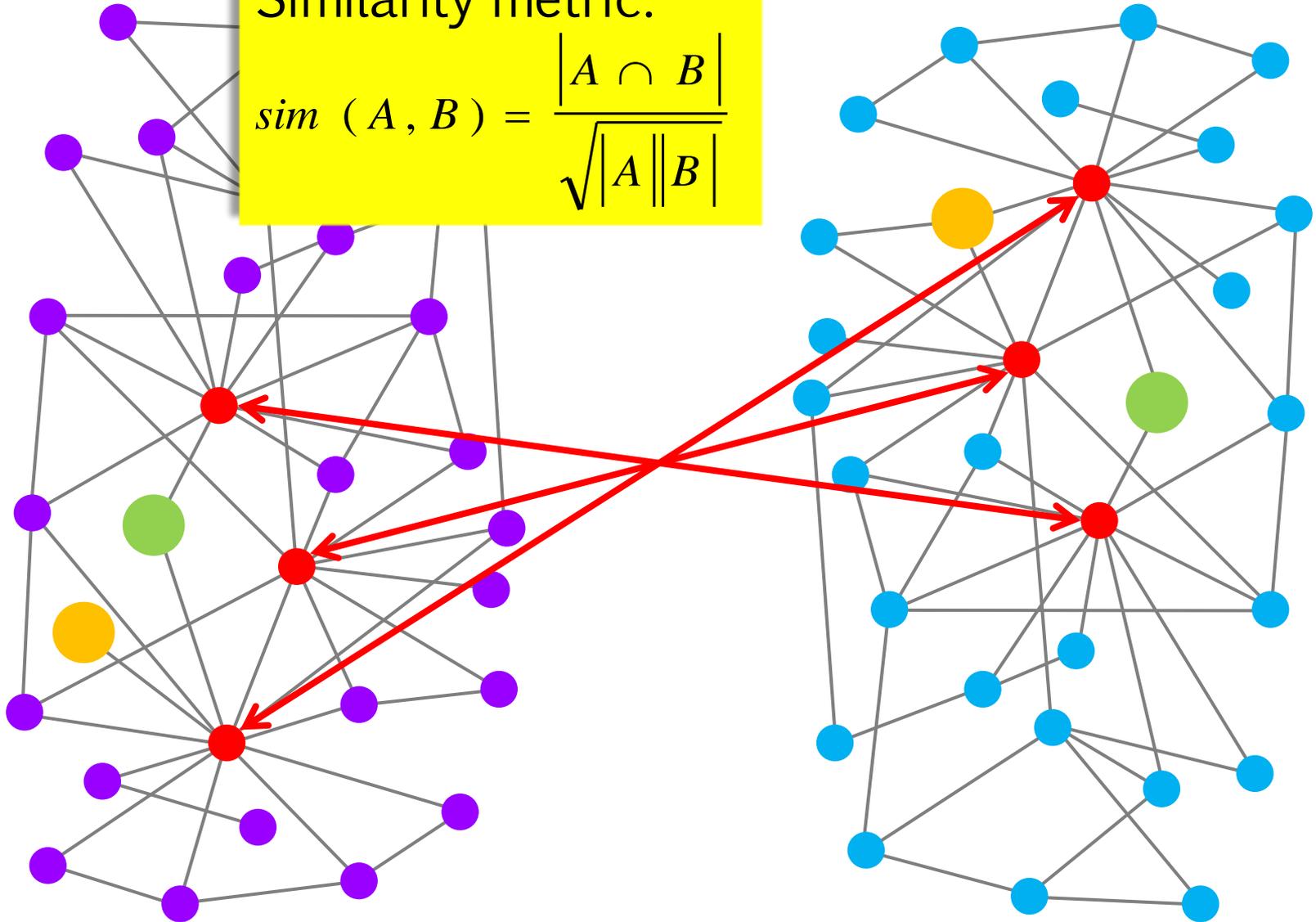


[A. Narayanan, V. Shmatikov, "De-anonymizing social networks", IEEE Symp. On Security and Privacy, 2009]

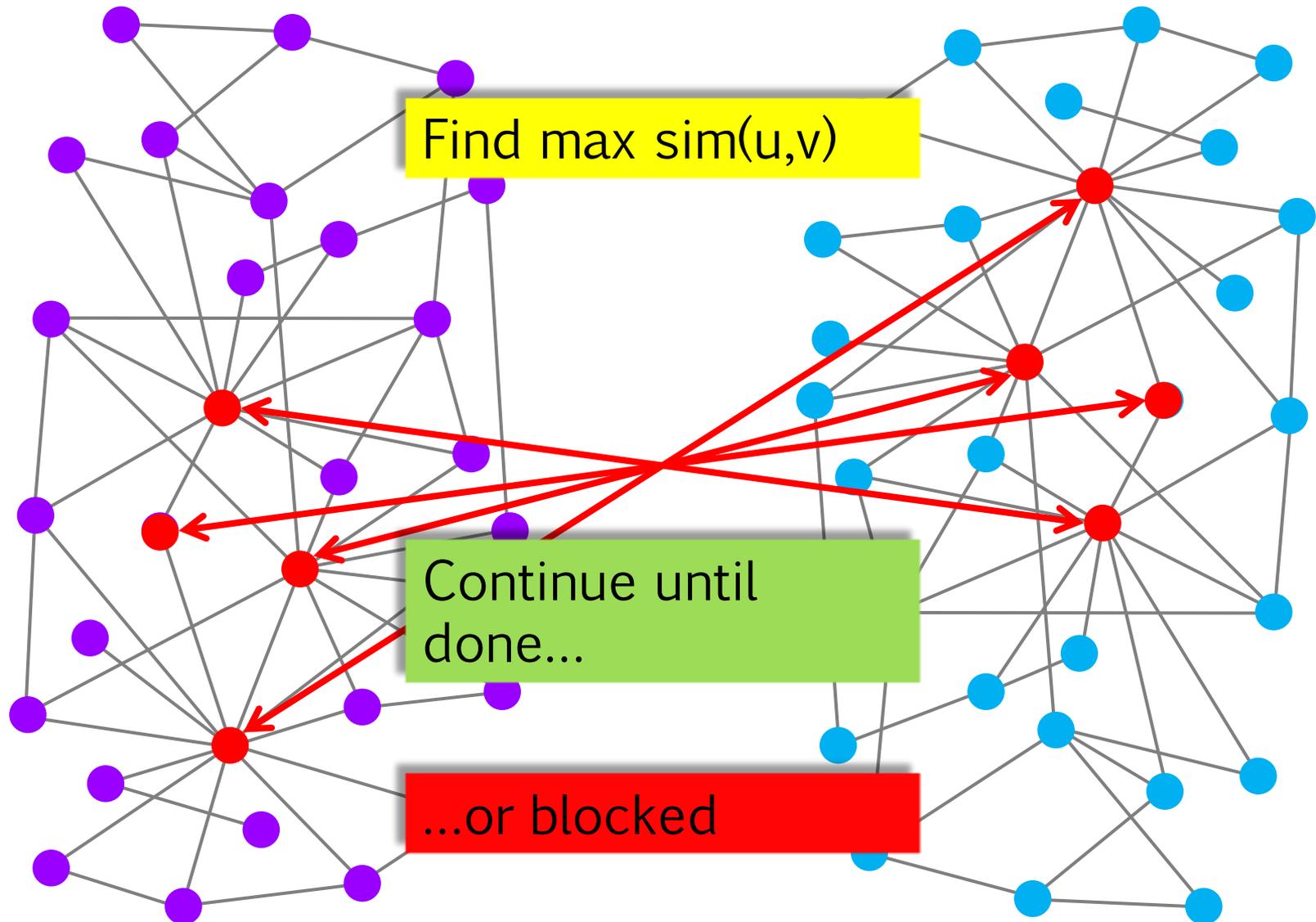
# Neighborhood Overlap Metric

Similarity metric:

$$\text{sim}(A, B) = \frac{|A \cap B|}{\sqrt{|A||B|}}$$



# Map Propagation



# Questions

- How many seeds are needed?
- Is there a phase transition?
- How efficiently can we match?
- Tuning parameters?

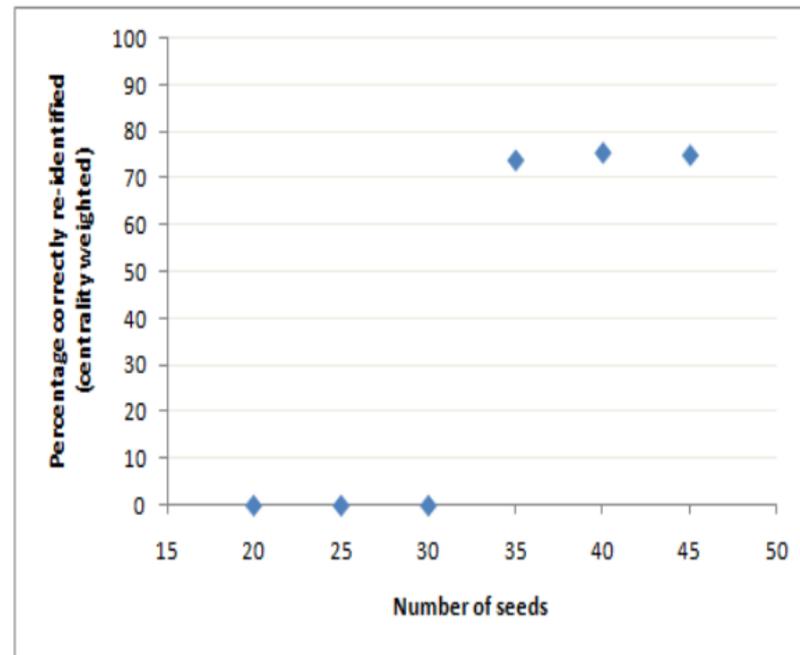
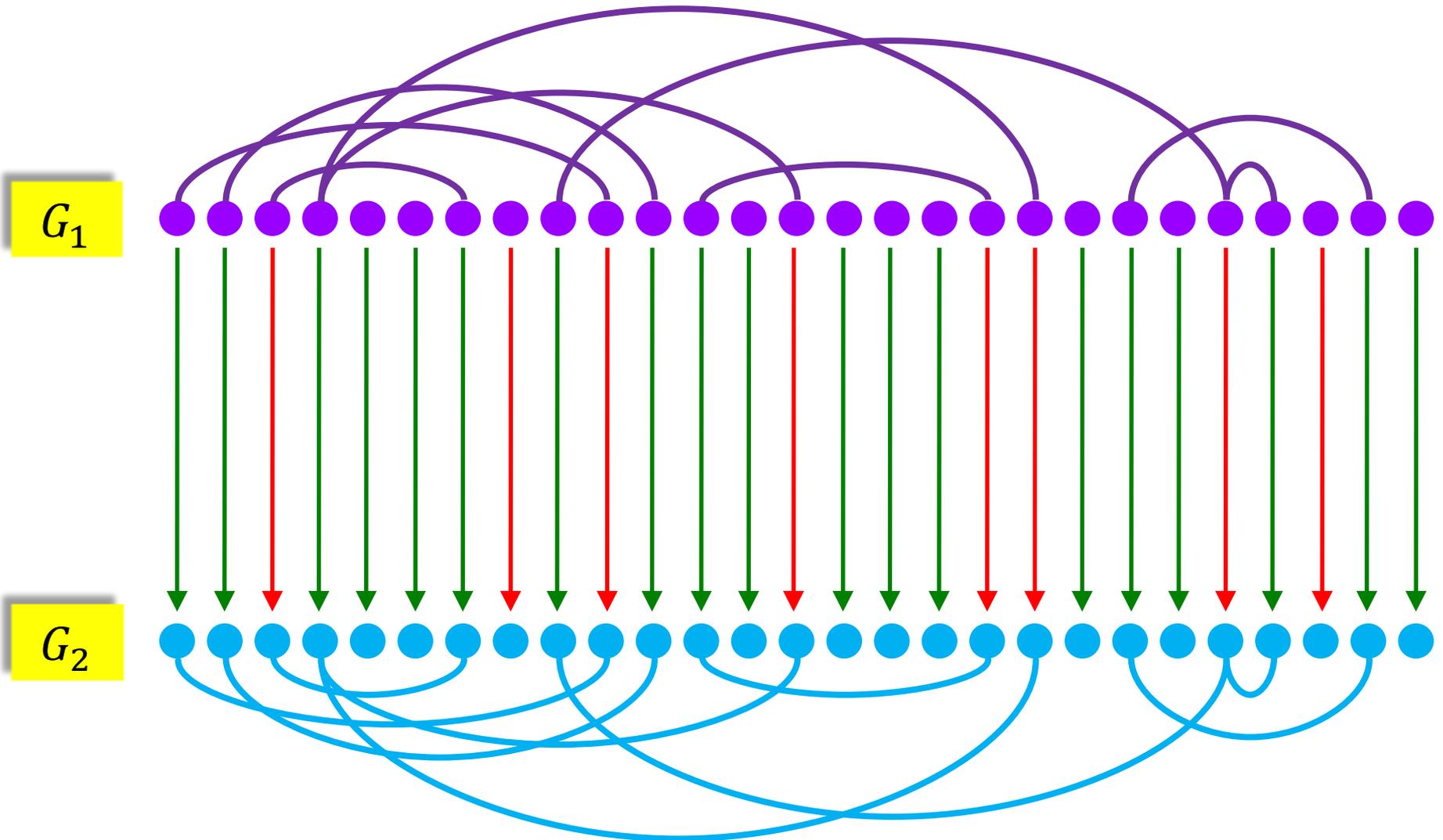


Figure 2. The fraction of nodes re-identified depends sharply on the number of seeds. Node overlap: 25%; Edge overlap: 50%

[A. Narayanan, V. Shmatikov, "De-anonymizing social networks", IEEE Symp. on Security and Privacy, 2009]

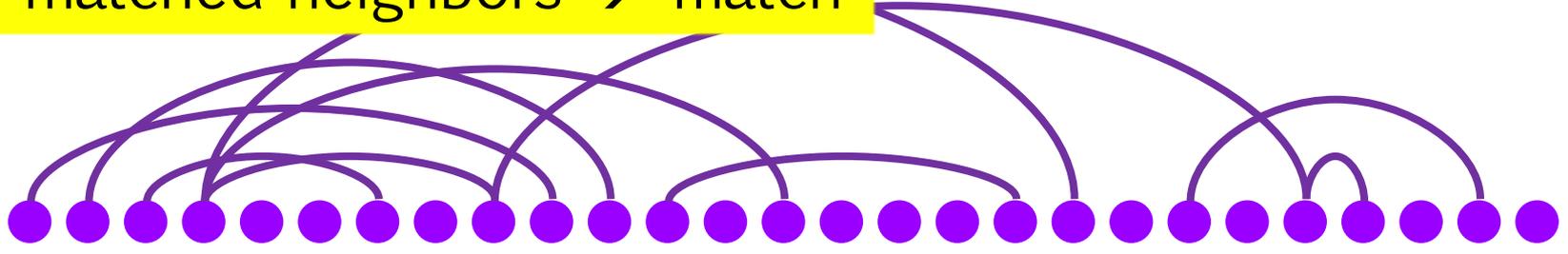
# Model: Two Similar Graphs + Seeds



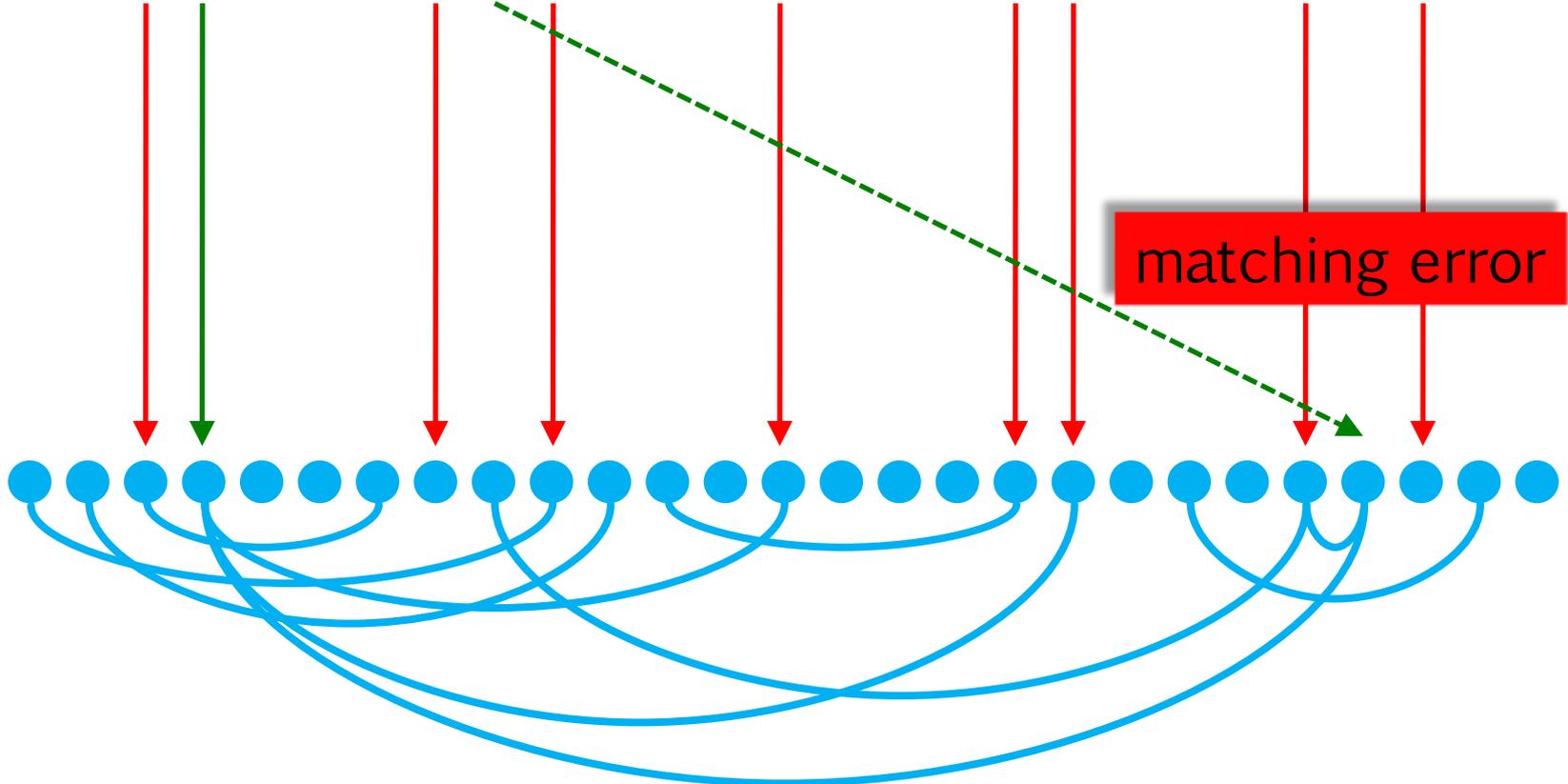
# Percolation Graph Matching Algorithm

If  $\geq r$  matched neighbors  $\rightarrow$  match

$G_1$

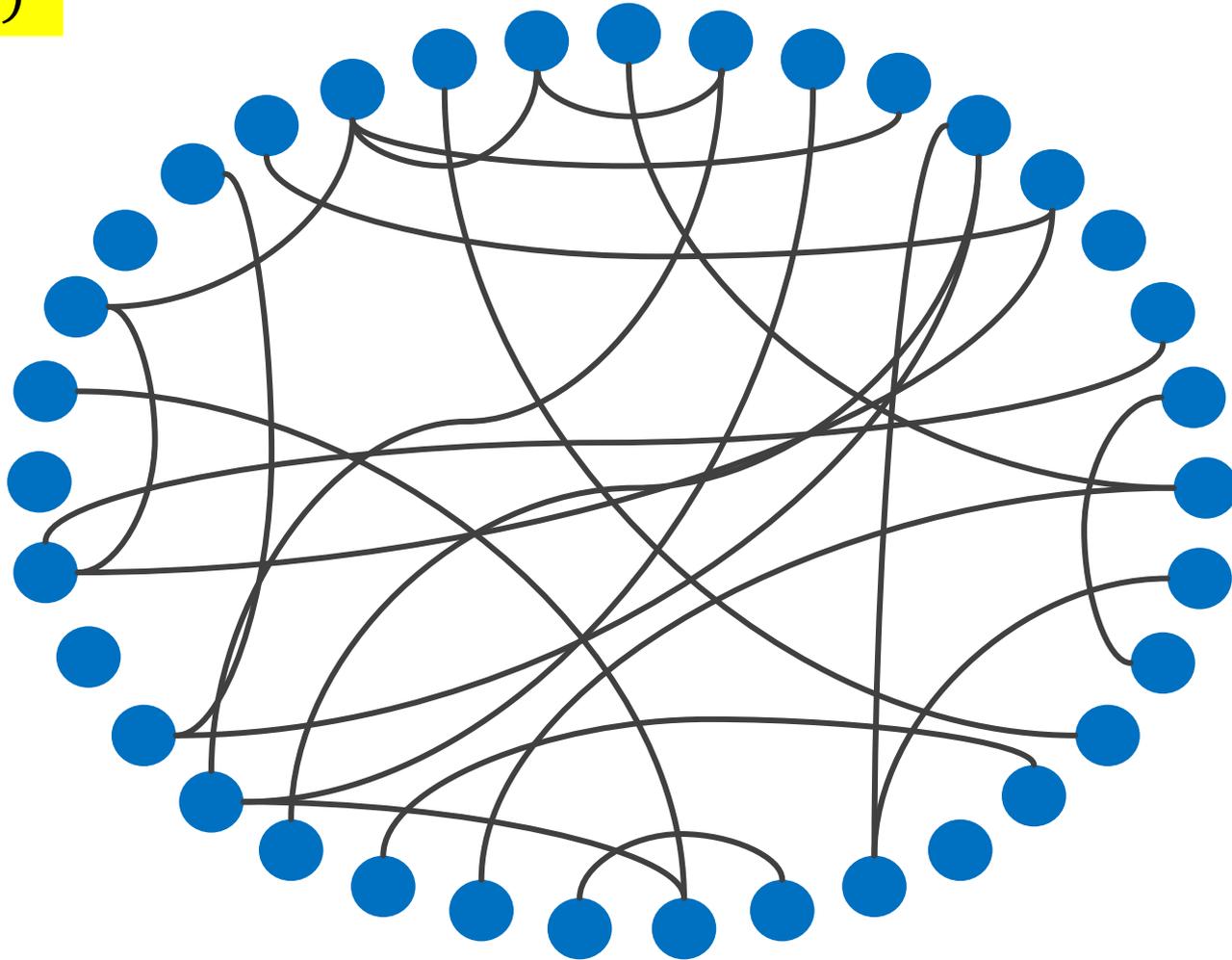


$G_2$

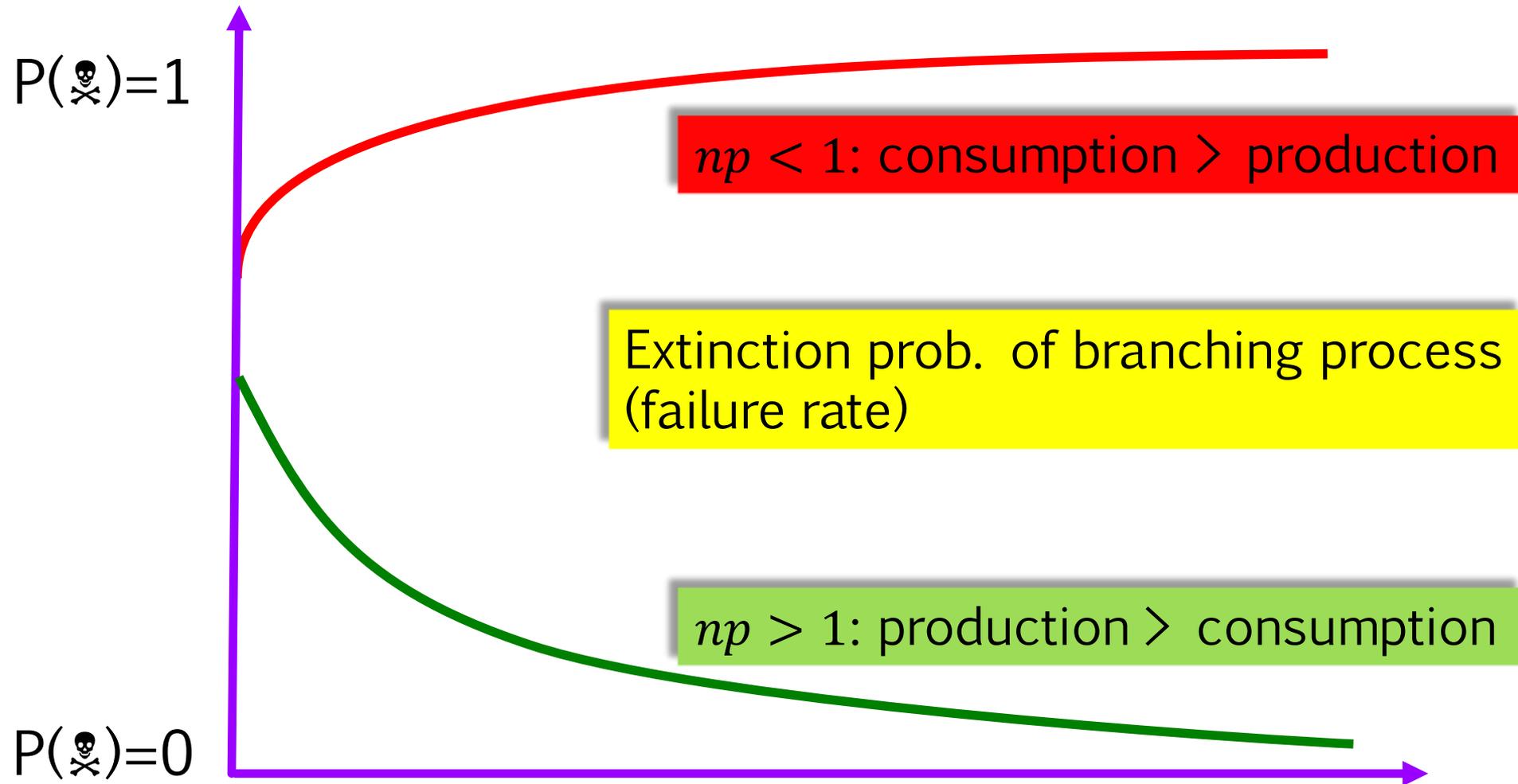


# Giant Component for $G(n, p)$

$G(n, p)$

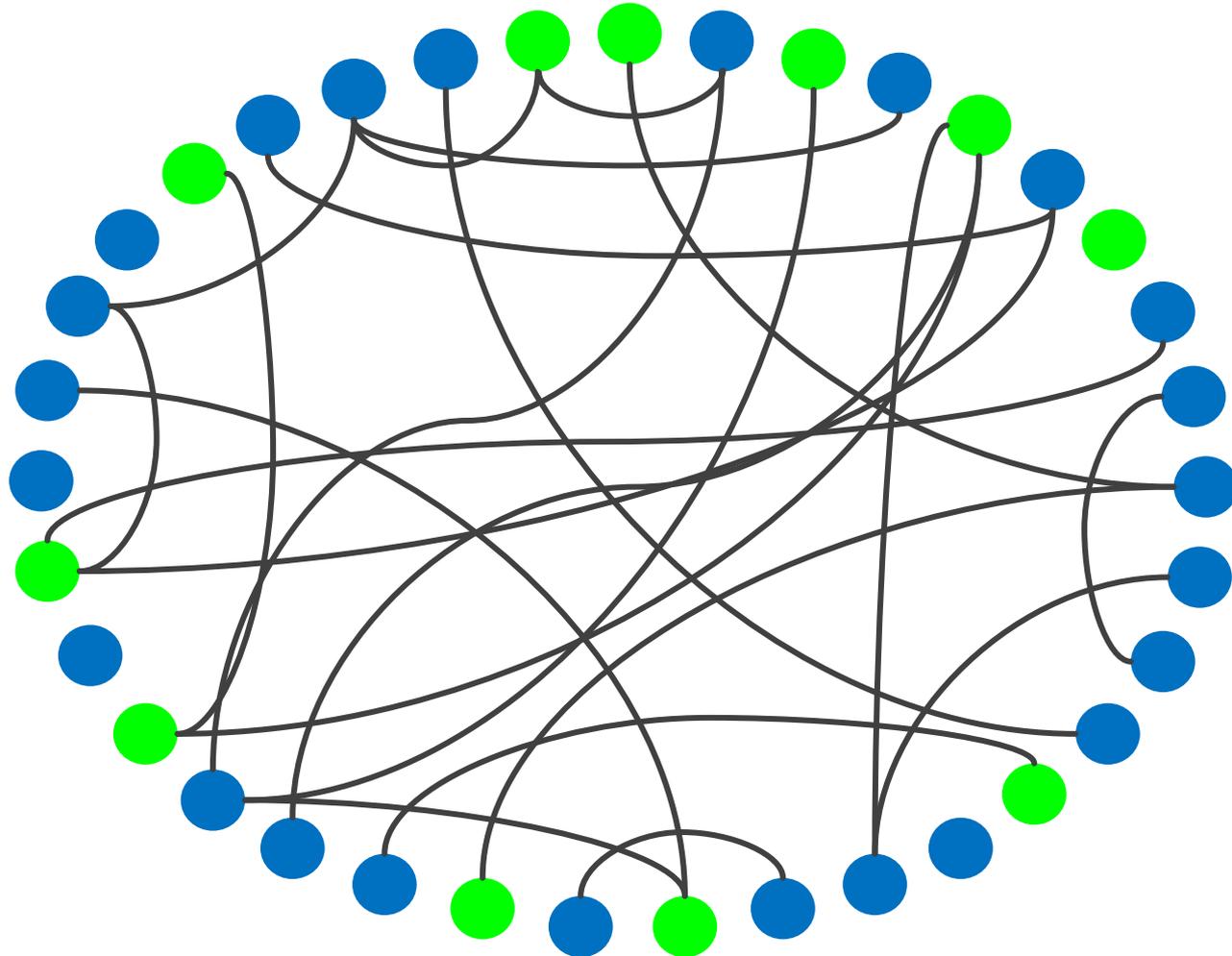


# Giant Component: Branching Process



# Bootstrap Percolation for $G(n, p)$

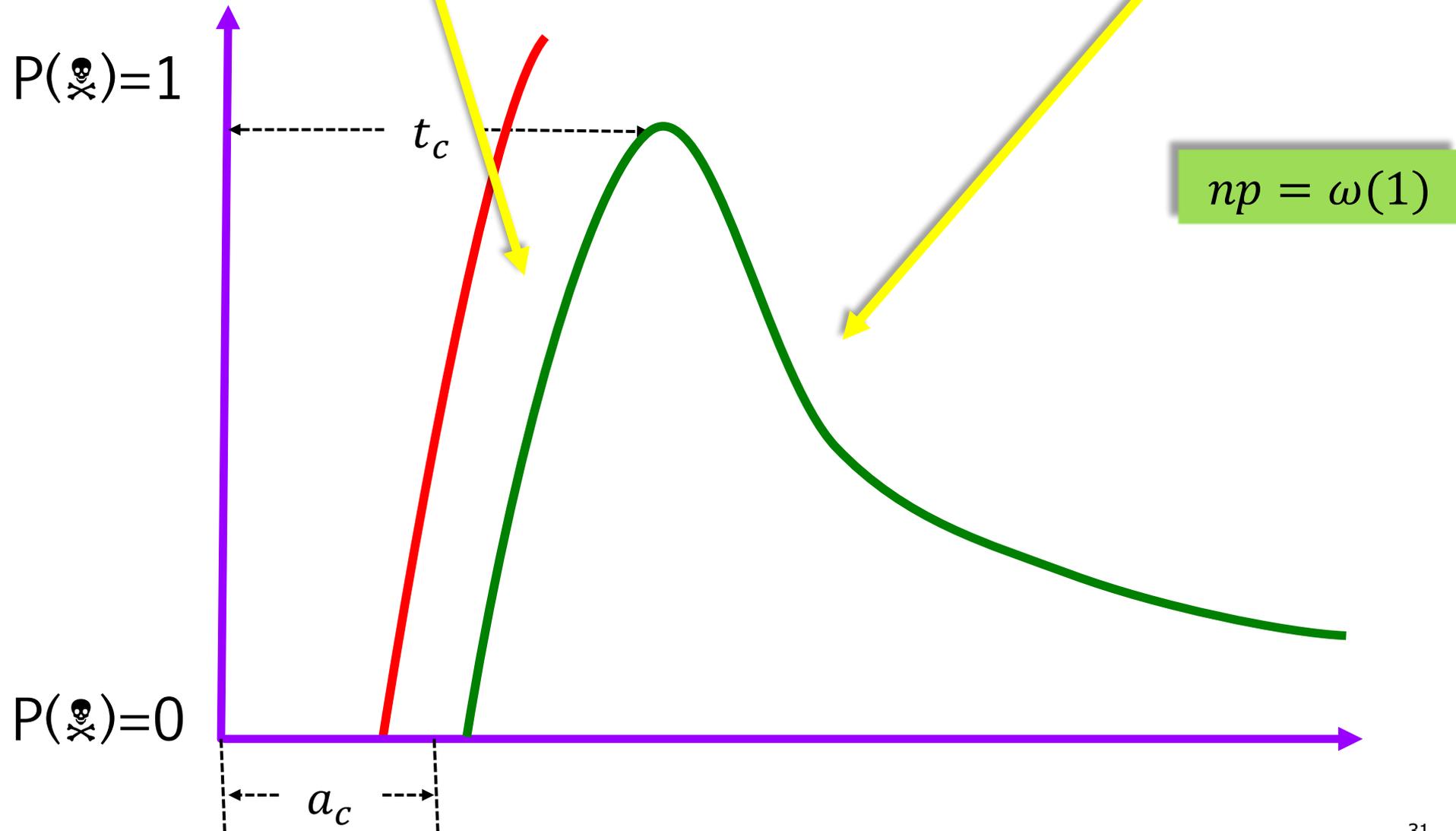
Activation from  $r$  neighbors



# Bottleneck in Bootstrap Percolation

consumption > production

production > consumption



# Results for Percolation Graph Matching

- **Theorem: phase transition in # seeds**

- For  $n^{-1} \ll ps \ll sn^{-\frac{1}{2} - \frac{3}{2r}}$ :

- If  $\frac{a}{a_c} \rightarrow \alpha < 1$ ,

final map is  $o(n)$  w.h.p.

- If  $\frac{a}{a_c} > \alpha > 1$ ,

final map is  $n - o(n)$  w.h.p.

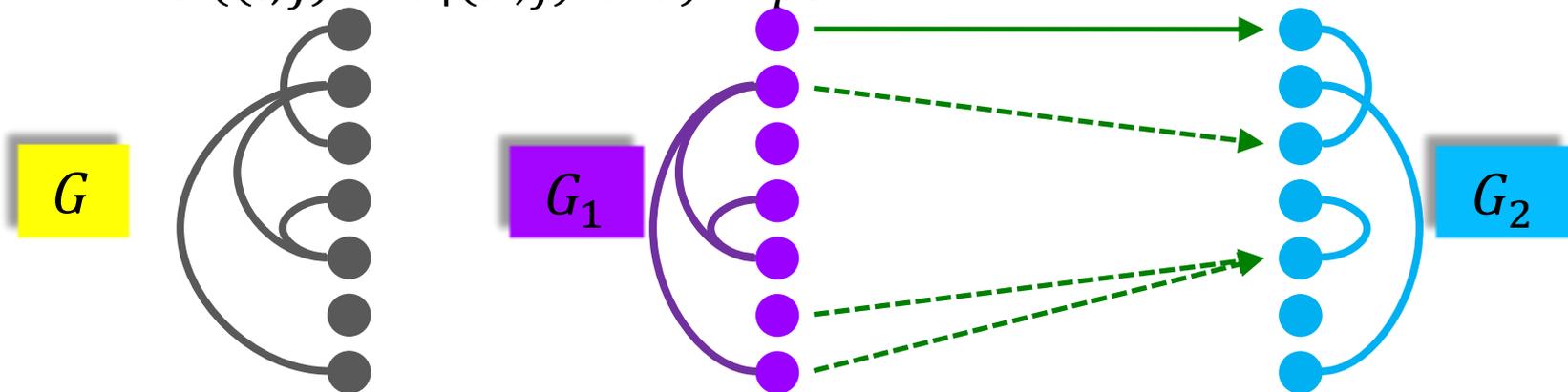
- **Seed set size threshold:**

- $a_c = (1 - r^{-1})t_c$

- $t_c = \left( \frac{(r-1)!}{n(ps^2)^r} \right)^{1/(r-1)}$

# Proof Sketch

- Bootstrap percolation in  $G(n, p)$ :
  - # credits of node  $i$  at time  $t$ : i.i.d. Binomials
- Percolation graph matching in  $G(n, p; s)$ 
  - # credits of pair  $(i, j)$  at time  $t$ : dependent, different Binomials
  - As long as no matching error so far, increments at  $t$
  - Different:  $(i, i) \sim \text{Ber}(ps^2)$ ,  $(i, j) \sim \text{Ber}((ps)^2)$
  - Dependent: for  $i, i', j$  all different:
    - $P((i, j)++) = (ps)^2$
    - $P((i, j)++ | (i', j)++) = ps$

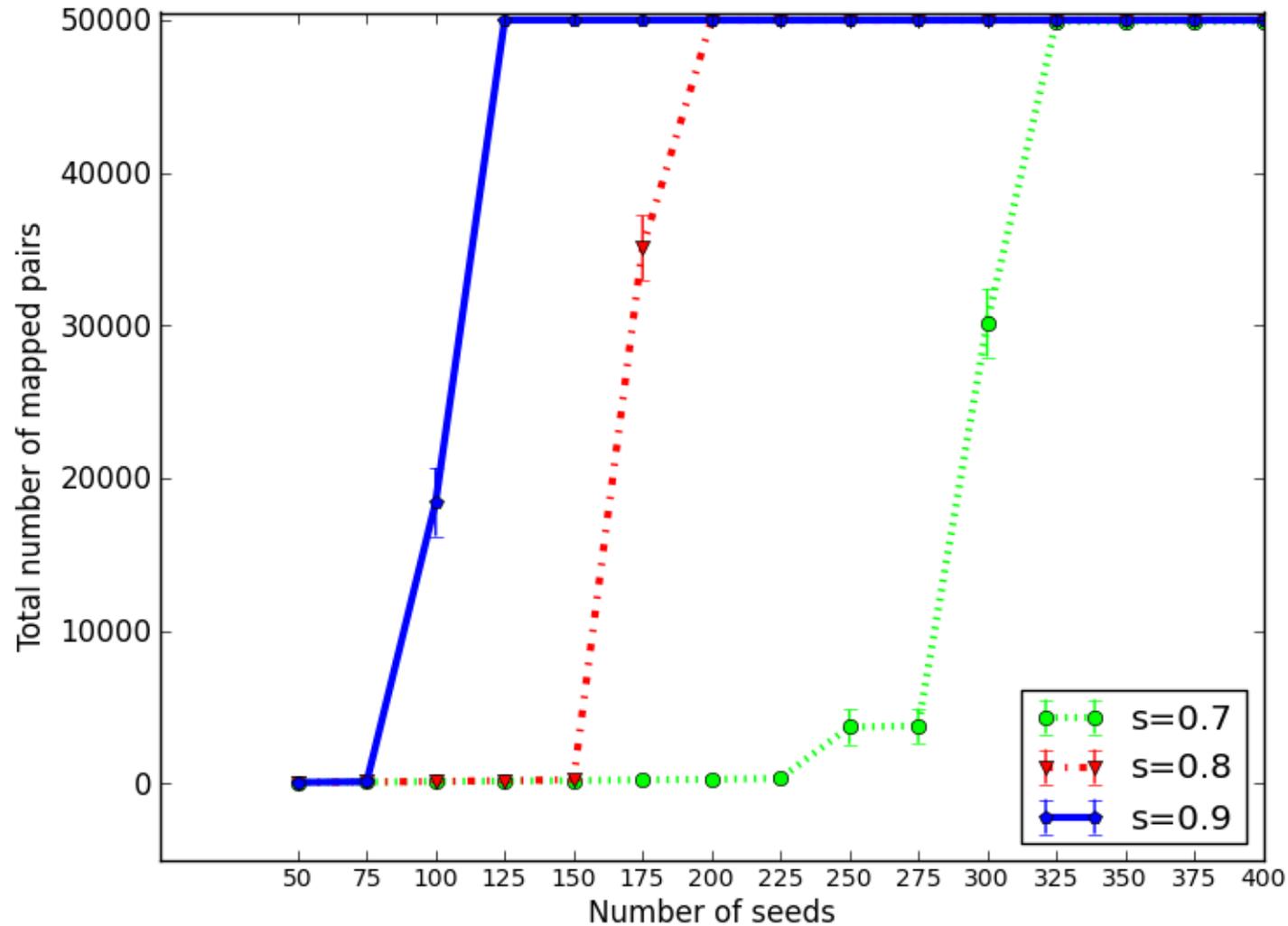


# Proof Sketch

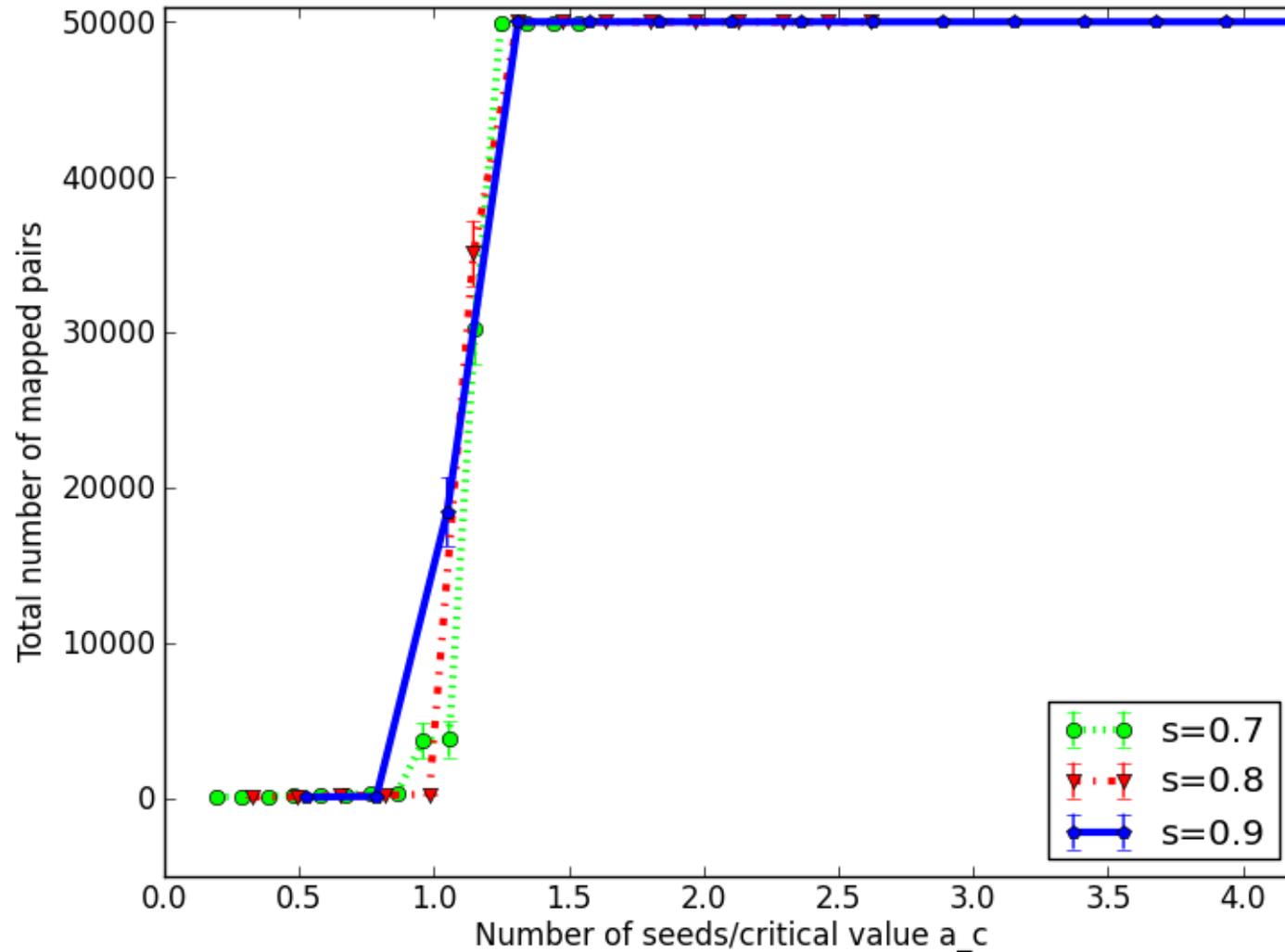
## ■ Approach:

- Focus on regime where  $X = \text{no bad pair } (i, j) \text{ get enough credits } (r) \text{ to be potentially matched}$
- True for  $ps \ll n^{-\frac{1}{2} - \frac{3}{2r}}$ 
  - Need to choose  $r$  large enough (sparse graphs:  $r \geq 4$ , otherwise higher)
- Conditional on  $X$ , only need to focus on good pairs  $(i, i)$
- Equivalence with bootstrap problem  $\rightarrow$  does it percolate?
  - Need to have  $n^{-1} \ll ps$
  - Need to have seed set size  $a > a_c$  large enough

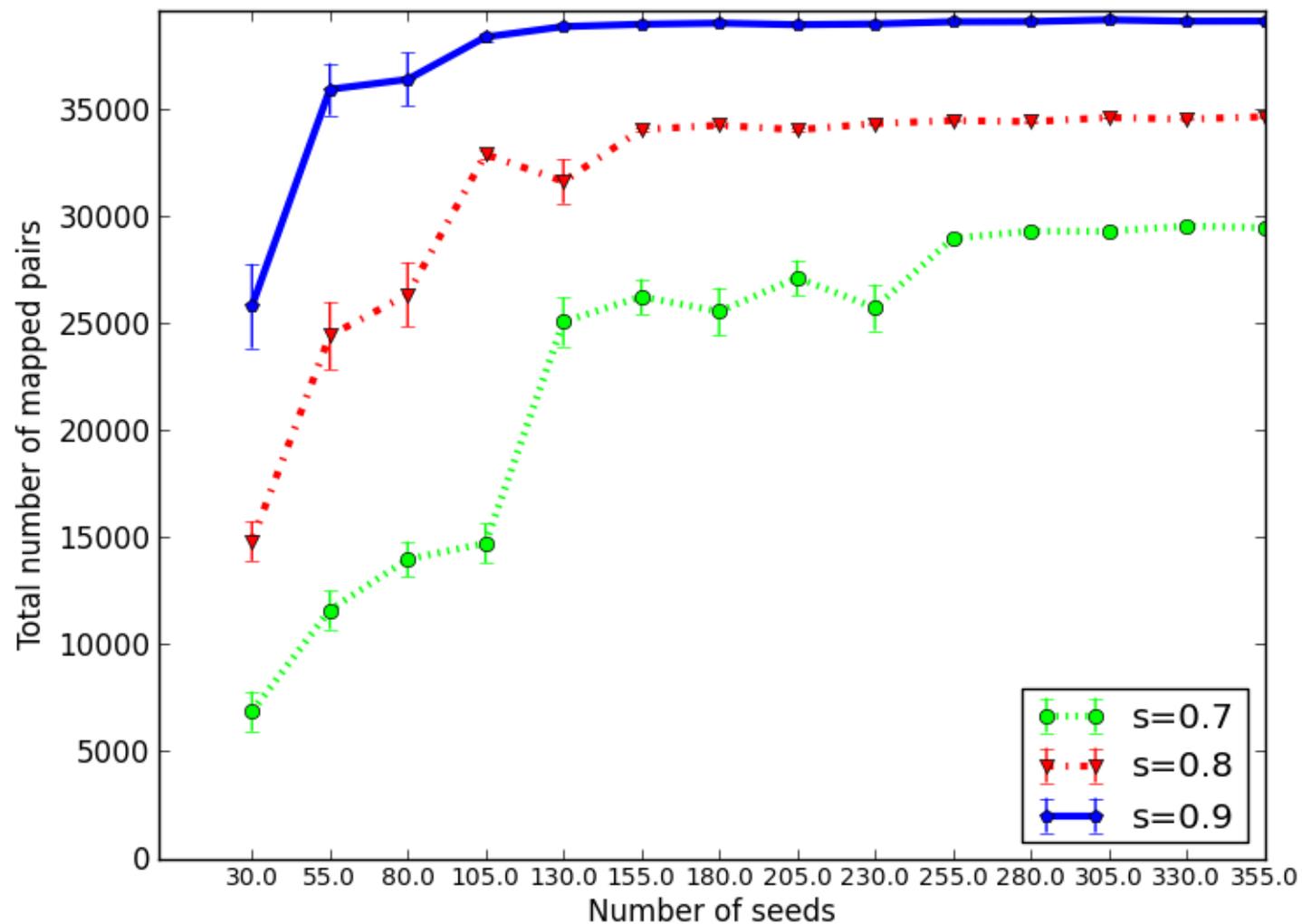
# Simulation of PGM with $G(n, p; s)$ Network



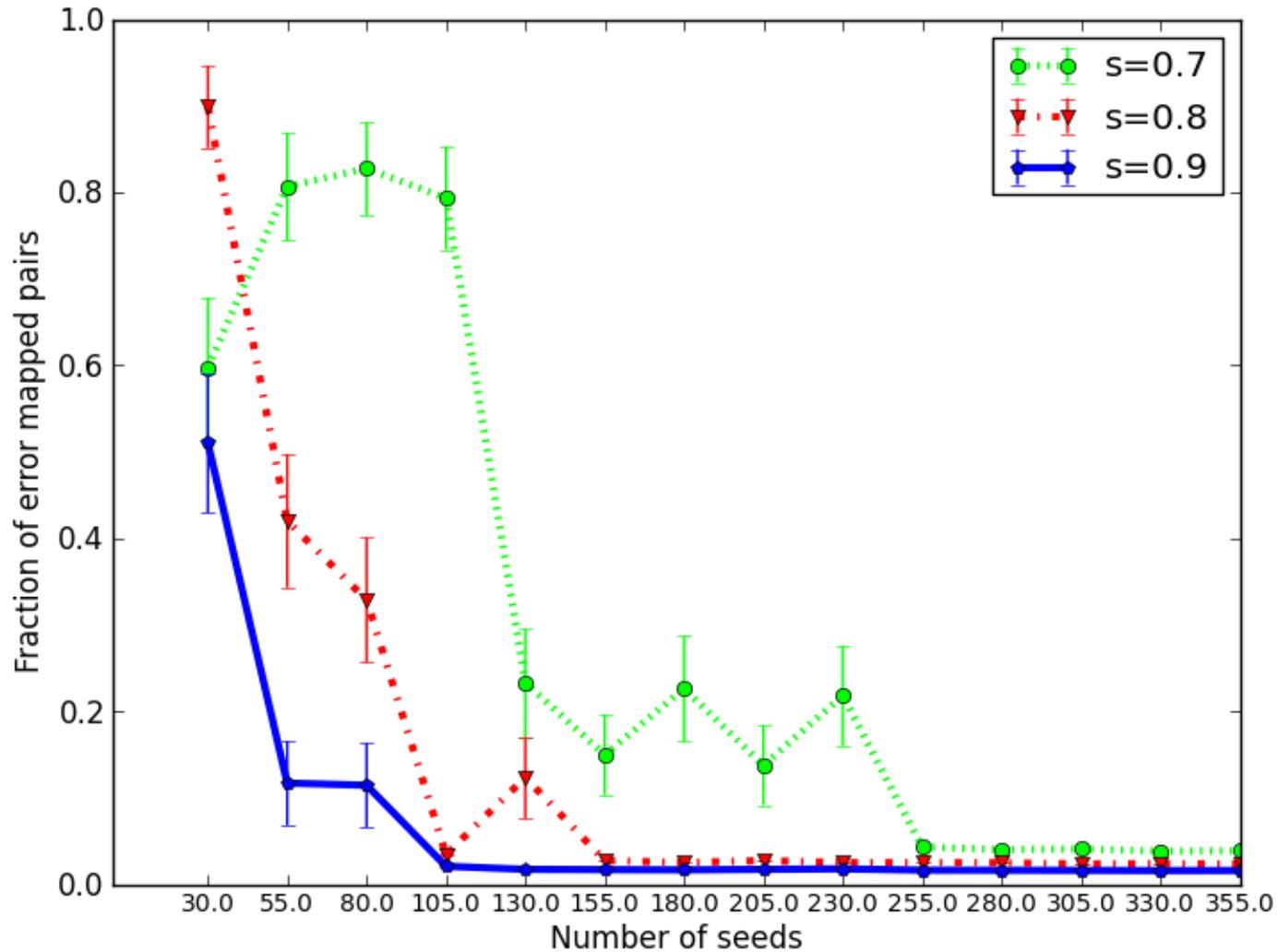
# Simulation of PGM with $G(n, p; s)$ Network



# Real Network: Slashdot Social Graph



# Real Network: Slashdot Social Graph

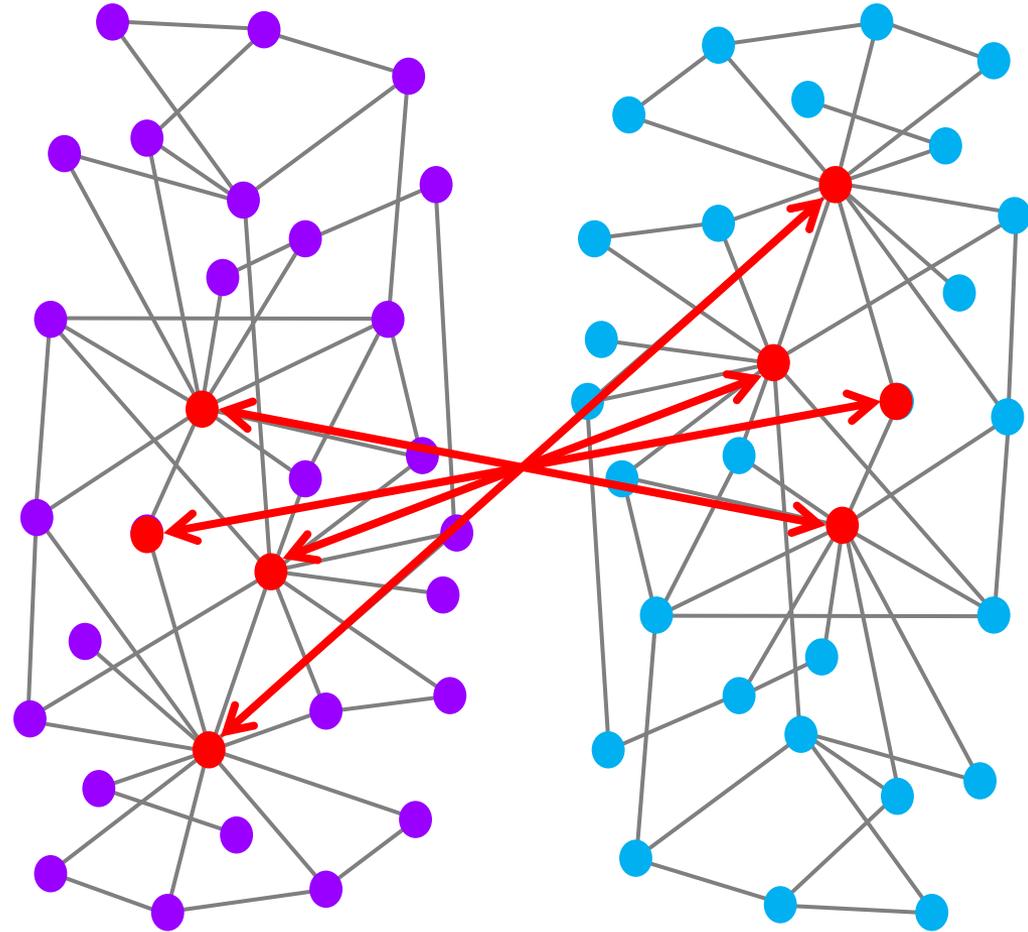


# Real networks

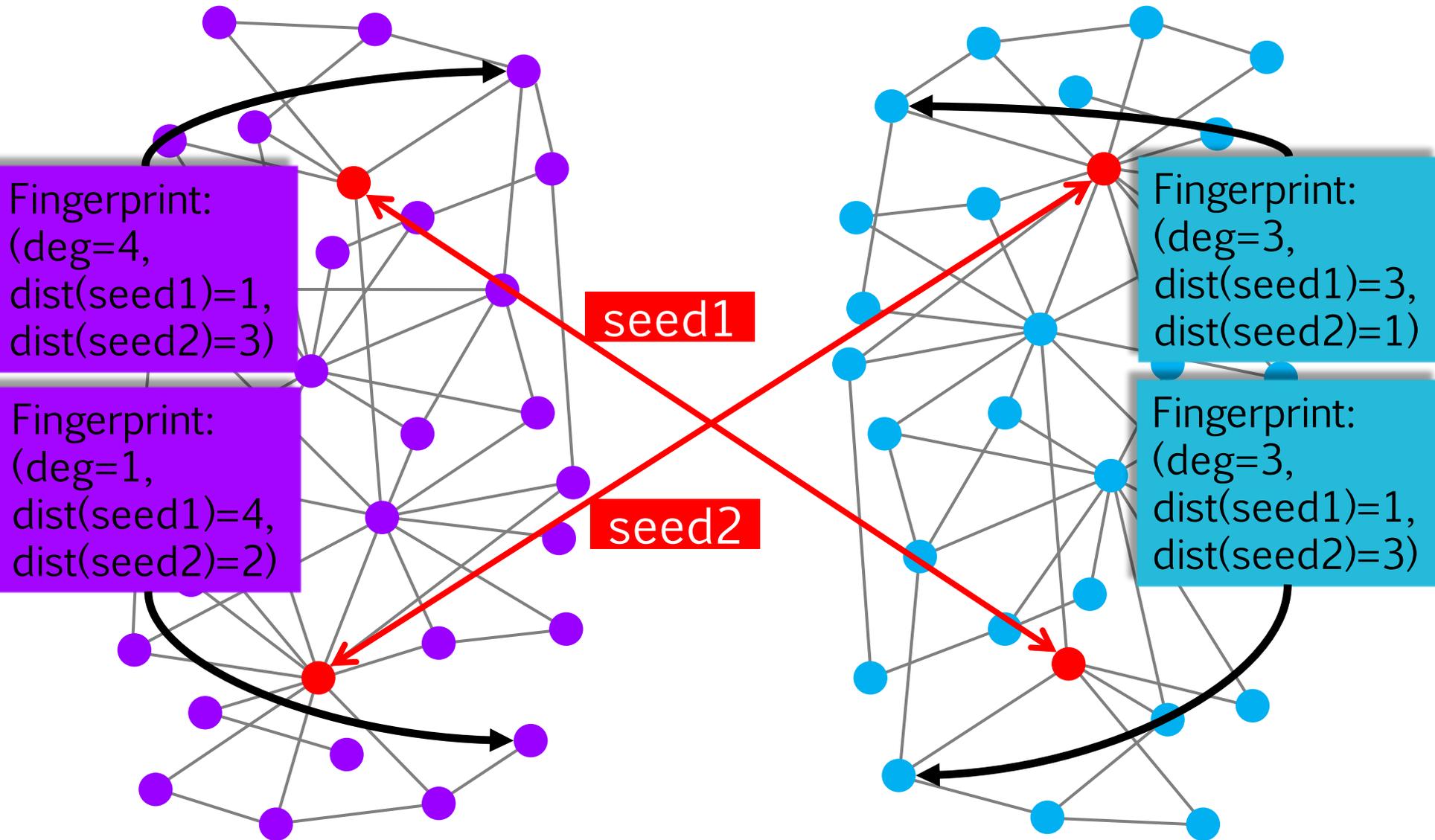
How to get started in practice

# Matching Algorithm and Sampling Model

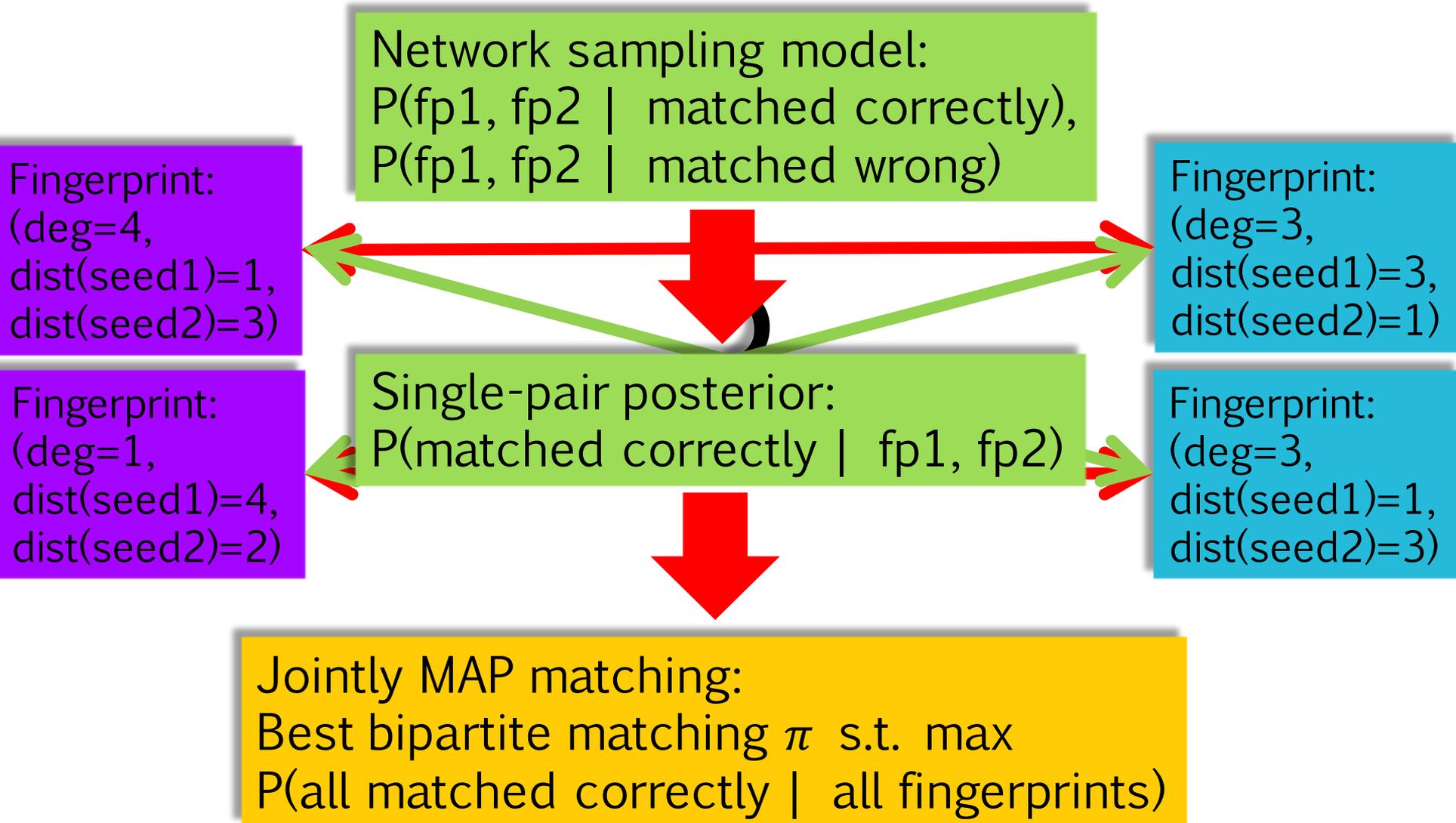
- **Question:**
  - Can similar idea inform algorithm design?
- **Wishlist:**
  - **Cold-start:** how to match without seeds?
  - **Sparse graphs:** how to avoid blocking?
  - **Error propagation:** how to correct mismatches?



# Matching Algorithms: Bayesian Framework



# Matching Algorithms: Bayesian Framework

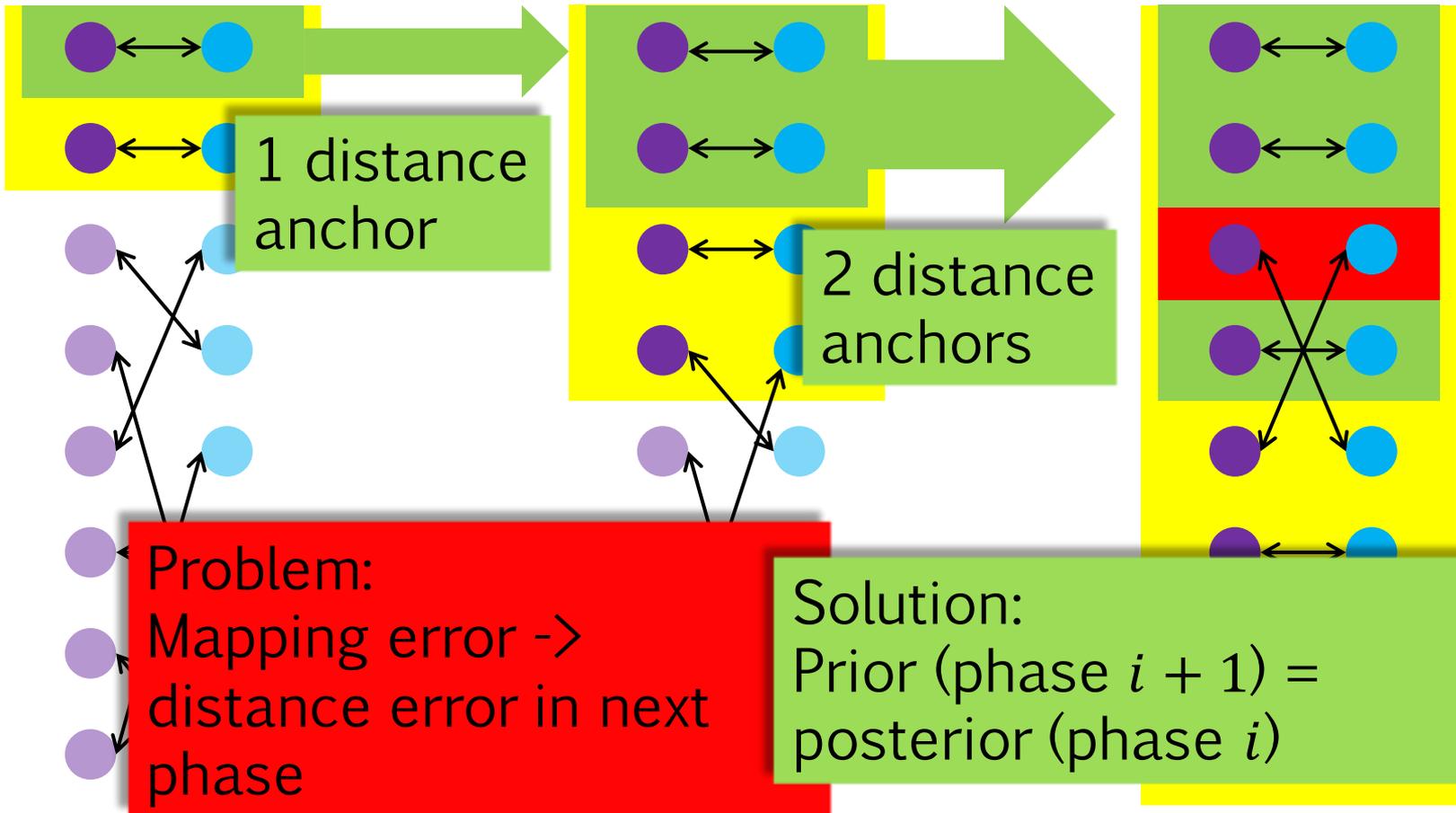


# Iterative Seedless Bayesian Matching

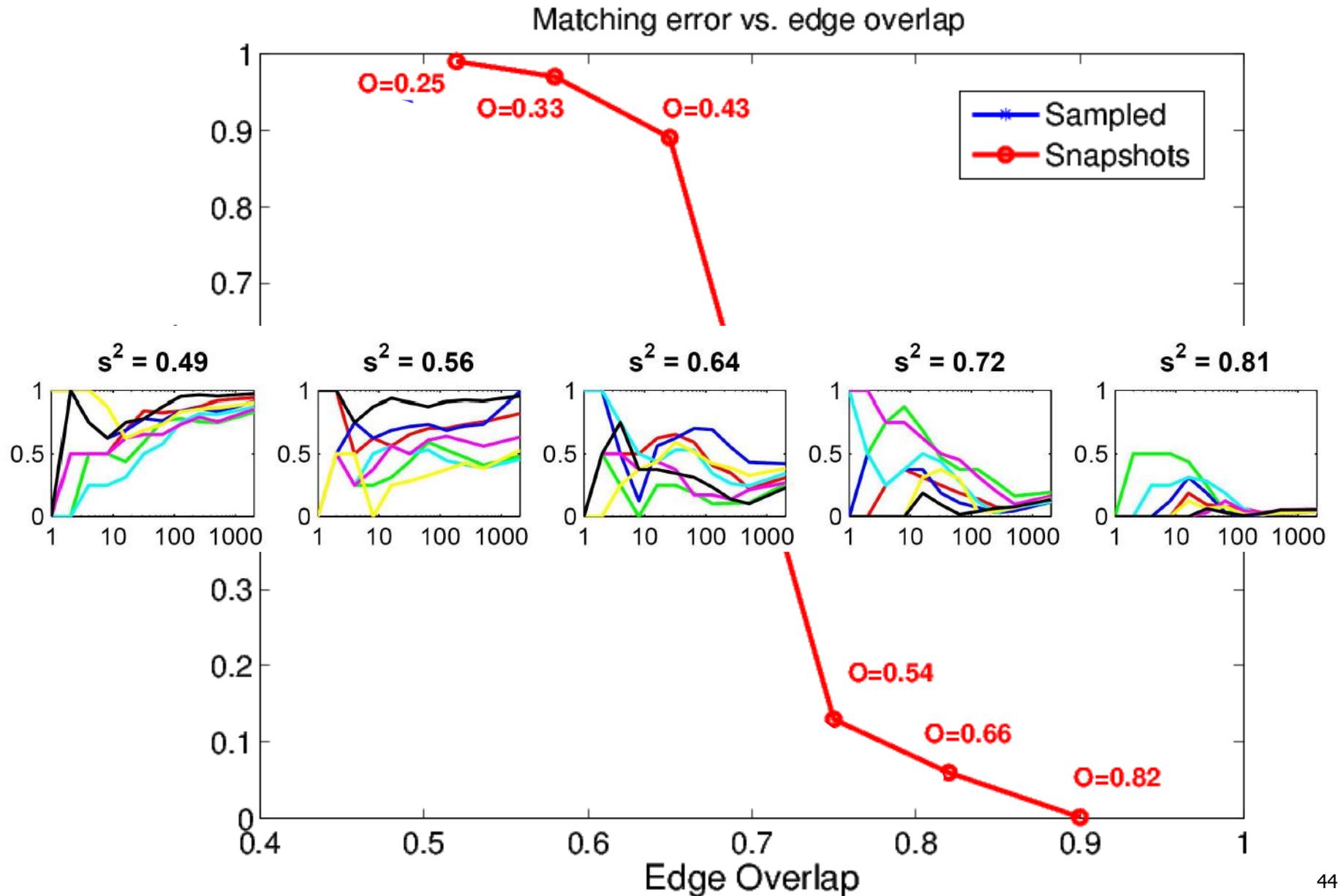
Phase 1:  
2 candidates

Phase 2:  
4 candidates

Phase 3:  
8 candidates



# Bayesian Seedless Matching: Performance



# Conclusion

- **Graph Matching:**

- Model as noisy graph isomorphism problem
- How much information in network structure?

- **Information-theoretic:**

- Matching is quite easy, benign growth of mean degree
- $G(n, p; s)$  model: no a-priori structure

- **Percolation Graph Matching from seeds**

- Phase transition in size of seed set  $\rightarrow$  hard to control, tune, predict
- Actually works very well in practice; parsimonious ( $r$ )

- **Finding seeds**

- Bayesian framework & heuristics
- Key idea: exploit known “couples” as references for new candidate pairs



# Thank you!

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Collaborators:  
Daniel R. Figueiredo,  
Pedram Pedarsani,  
Lyudmila Yartseva



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FÉDÉRALE DE LAUSANNE