Collaborative Interference Management: When are the “Conventional Schemes” Optimal?

Salman Avestimehr
Cornell University

Joint work with Chunhua Geng (UCI), Navid NaderiAlizadeh (Cornell), and Syed Jafar (UCI)

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Interference Management Dichotomy

when to use each scheme?

conventional approaches

- power control with treat-interference-as-noise,
  interference avoidance, etc.

modern approaches

- interference alignment,
  interference neutralization, etc.

amount of feedback (to acquire CSI)
When is “power control + treat-interference-as-noise” optimal?

- **conventional approaches**
  - power control with treat-interference-as-noise, interference avoidance, etc.

- **new approaches**
  - interference alignment, interference neutralization, etc.

amount of feedback (to acquire CSI)
Setting

- K-user flat fading Gaussian interference channel

- Received signal at receiver j:

\[ Y_j(t) = \sum_{i=1}^{K} h_{ji} X_i(t) + Z_j(t) \]

- Channel from \( T_i \) to \( R_j \):

\[ h_{ji} = \sqrt{P_{\alpha_{ji}}} e^{\theta_{ji}} \]

channel strength (in dB, some base \( P>1 \))

\( \mathcal{CN}(0, 1) \) power constraint
Rate-Region of Treat-Interference-as-Noise (TIN)

• $T_i$ uses power of $P^{r_i}$ (for some $r_i \leq 0$)

• Signal-to-interference-plus-noise ratio (SINR) at $R_i$

$$\text{SINR}_i = \frac{P^{\alpha_{ii}} \times P^{r_i}}{1 + \sum_{j \neq i} P^{\alpha_{ij}} \times P^{r_j}}$$

• Rate region of TIN

$$\mathcal{R}_{\text{TIN}} = \bigcup_{r_i \leq 0} \{(R_1, \ldots, R_K) : R_i = \log(1 + \text{SINR}_i)\}$$

• Generalized Degrees of Freedom (GDoF) region of TIN

$$\mathcal{D}_{\text{TIN}} = \lim_{P \to \infty} \frac{\mathcal{R}_{\text{TIN}}}{\log P}$$

$$Y_j(t) = \sum_{i=1}^{K} h_{ji} X_i(t) + Z_j(t)$$
What is known about the optimality of TIN?

\[ R_{\text{TIN}} = \log\left(1 + \frac{P}{1 + P^\alpha}\right) \]
\[ \approx (1 - \alpha) \log(P) \]

\[ \text{INR}_{dB} \leq \frac{1}{2} \times \text{SNR}_{dB} \]
How about in general?

• Not much is known about its optimality (except for some symmetric and very low interference regimes)

• TIN region is hard to analyze analytically
  • Includes a (hard) optimization problem over \( r_i \)'s (i.e., power allocation, e.g. Foschini-Milijanic 1993, Tan-Chiang-Srikant 2013)
  • Can be characterized through solving a sequence of GP’s (Mahdavi et al 2008)
  • Non-explicit and non-convex region in general

• Very few general bounds on the capacity region of K-user IC

\[
\mathcal{R}_{\text{TIN}} = \bigcup_{r_i \leq 0} \{(R_1, \ldots, R_K) : R_i = \log(1 + \text{SINR}_i)\}
\]
Main Result

**Theorem:** In a $K$-user interference channel, if

$$\alpha_{ii} \geq \max_{j:j \neq i} \alpha_{ji} + \max_{k:k \neq i} \alpha_{ik}, \quad \forall i, j, k \in \{1, 2, \ldots, K\}$$

- TIN achieves the capacity region within a constant gap of $\log_2(3K)$ bits,
- TIN region is approximated by a polyhedron.

- In words, the condition is

  "at each user, the desired channel strength is at least the sum of the strengths of the strongest interference from this user and the strongest interference to this user"

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- Capacity approximation in a specific regime
- Underlying structure of the TIN rate-region
- A general condition for when acquiring additional CSI (e.g., phase) does not worth it
Key Challenges

**Theorem:** In a $K$-user interference channel, if

“at each user, the desired channel strength is at least the sum of the strengths of the strongest interference from this user and the strongest interference to this user”

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- Explicit characterization of the TIN rate-region.
- Matching outerbounds.
Step 1: polyhedral relaxation of TIN

- Rate of each user (for a given power allocation):
  \[ d_i = \frac{R_i}{\log P} \approx \alpha_{ii} + r_i - \max_{j \neq i}(\alpha_{ij} + r_j)^+ \]

- Polyhedral relaxation of TIN:

\[ \bigcup r_i \leq 0 \begin{cases} 
  d_i \leq \alpha_{ii} + r_i \\
  d_i \leq (\alpha_{ii} + r_i) - (\alpha_{ij} + r_j), \quad j \neq i 
\end{cases} \]

\[ R_i = \log \left( 1 + \frac{P^\alpha_{ii} + r_i}{1 + \sum_{j \neq i} P^{\alpha_{ij} + r_j}} \right) \approx P^{\max_{j \neq i}(\alpha_{ij} + r_j)^+} \]
Step 2: Relation to Potential Functions

- Polyhedral TIN:
  \[
  \bigcup_{r_i \leq 0} \begin{cases} 
  d_i \leq \alpha_{ii} + r_i \\
  d_i \leq (\alpha_{ii} + r_i) - (\alpha_{ij} + r_j), & j \neq i 
  \end{cases}
  \]

  - It is the set of all \((d_1, \ldots, d_K)\) for which there exists \((r_1, \ldots, r_K)\) satisfying
    - \(r_i \leq 0\)
    - \(-r_i \leq \alpha_{ii} - d_i\)
    - \(r_j - r_i \leq \alpha_{ii} - \alpha_{ij} - d_i\)

  - It is the set of all \((d_1, \ldots, d_K)\) for which there exists a potential function on

  \(r : V \to \mathbb{R}\), is a potential function on \(G = (V, E, w)\), if
  \[r(j) - r(i) \leq w(i, j), \quad \forall i, j \in V\]
Step 3: Potential Theorem

- **Theorem [Hoffman]:** There exists a potential function for a directed graph $D$ if and only if all directed loops have non-negative weights.

- Therefore, $(d_1, \ldots, d_K)$ is in polyhedral TIN if and only if

$$
\sum_{\ell=0}^{m-1} d_{i\ell} \leq \sum_{\ell=0}^{m-1} (\alpha_{i\ell i\ell} - \alpha_{i\ell i\ell+1})
$$
Recap

- Polyhedral TIN: $\cup r_i \leq 0 \left\{ \begin{align*}
    d_i &\leq \alpha_{ii} + r_i \\
    d_i &\leq (\alpha_{ii} + r_i) - (\alpha_{ij} + r_j), \quad j \neq i
\end{align*} \right.$

Figure 2: (a) A 3-user interference channel, where the value on each link is equal to its channel strength level, and (b) The GDoF region of this network, which is a convex polyhedron and can be achieved by TIN.

3.1 Polyhedral Relaxation of TIN

In the first step toward proving Theorem 1, we introduce a polyhedral version of the TIN scheme. Ignoring the first max term in (8) changes the scheme to a relaxed version, which we call the polyhedral TIN scheme. With this modification, the GDoF achieved by user $i$ will be $d_i = \sum_{j \neq i} (\alpha_{ij} + r_j)$, and we denote the achievable GDoF region via polyhedral TIN by $P$. In general, comparing (8) and (12) shows that this modification can only shrink the achievable GDoF region of TIN. However, as we will show in the following, under the condition (9), the above relaxation incurs no loss in the GDoF region of TIN. In other words, when the condition (9) is satisfied, the TIN region $P^*$ is equal to the polyhedral TIN region $P$. From (12), the polyhedral TIN region $P$ can be characterized by a number of linear inequalities, which, as we will see, significantly contributes to understanding the TIN region $P^*$. In fact, $P$ is the set of all $K$-tuples $(d_1, \ldots, d_K)$ s.t.

$$0 \leq d_i \leq \alpha_{ii}$$

$$(d_1, \ldots, d_K) \text{ s.t. } \left\{ \begin{align*}
    d_i &\leq \alpha_{ii} + r_i \\
    d_i &\leq (\alpha_{ii} + r_i) - (\alpha_{ij} + r_j), \quad j \neq i
\end{align*} \right.$$
Step 4: Matching Outerbounds

- **Lemma**: Consider a K user interference channel, such that

\[
\alpha_{ii} \geq \max_{j:\ j \neq i} \alpha_{ji} + \max_{k:\ k \neq i} \alpha_{ik}, \quad \forall i, j, k \in \{1, 2, \ldots, K\}
\]

then any \((d_1, \ldots, d_K)\) in the GDoF region satisfies

\[
0 \leq d_i \leq \alpha_{ii} \\
\sum_{\ell=0}^{m-1} d_{i\ell} \leq \sum_{\ell=0}^{m-1} (\alpha_{i\ell i_{\ell+1}} - \alpha_{i\ell i_{\ell+1}})
\]
**Main Result**

**Theorem:** In a $K$-user interference channel, if

“at each user, the desired channel strength is at least the sum of the strengths of the strongest interference from this user and the strongest interference to this user”

- TIN achieves the capacity region within a constant gap of $\log_2(3K)$ bits,
- TIN region is approximated by a polyhedron.

\[ (d_1, \ldots, d_K) \text{ s.t. } \left\{ \begin{array}{l} 0 \leq d_i \leq \alpha_{ii} \\ \sum_{\ell=0}^{m-1} d_{i\ell} \leq \sum_{\ell=0}^{m-1} (\alpha_{i\ell} i_{i\ell} - \alpha_{i\ell} i_{i\ell+1}) \end{array} \right. \]

Rate-Region of TIN in General

Theorem: In a $K$-user interference channel, the rate region achievable by TIN is within $\log_2(3K)$ bits of

$$\mathcal{P}^* = \bigcup_{S \subseteq \{1, \ldots, K\}} \mathcal{P}_S$$

where $\mathcal{P}_S$ is the polyhedral TIN region when the users in $S$ are silent.
Necessary and Sufficient Conditions

• Is the condition both necessary and sufficient for optimality of TIN (to within a constant gap)?

\[ \alpha_{ii} \geq \max_{j:j \neq i} \alpha_{ji} + \max_{k:k \neq i} \alpha_{ik}, \quad \forall i, j, k \in \{1, 2, \ldots, K\} \]

**Conjecture:** In a $K$-user interference channel, TIN is constant-gap optimal if and only if

\[ \alpha_{ii} \geq \max_{j:j \neq i} \{\alpha_{ji}\} + \max_{k:k \neq i} \{\alpha_{ik}\}, \quad \forall i, j, k \in \{1, 2, \ldots, K\} \]

except for a set of measure-zero gains.

However, once the phases are perturbed, TIN isn’t constant-gap optimal anymore.
Successive Interference Cancellation

- Rate region can be increased, using
  - Superposition coding
  - Successive interference cancellation
  - Treat-interference-as-noise
- When would that be optimal?

*Y. Zhao, C. Tan†, S. Avestimehr‡, S. Diggavi and G. Pottie “On the Sum-Capacity with Successive Decoding in Interference Channels”, IT 2012.*
Concluding remarks

- Established general conditions for (approximate) optimality of power control + treat-interference-as-noise
  - A general condition for when acquiring additional CSI does not worth it
  - Underlying structure of the TIN rate-region
- Network-level analysis of the capacity gains?
- How much can one go beyond TIN when CSI is only the channel gain, or channel-gain + some local phase?
Questions?